NCSE MATHEMATICS PAPER 2
YEAR 2010
Section I

1. (a) **Required to calculate:** The exact value of $1\frac{1}{2}+\left(\frac{1}{4}\times 1\frac{3}{5}\right)$

Calculation:
Working first, the part of the question that is written within brackets:

$$\frac{1}{4}\times 1\frac{3}{5}=\frac{1}{4}\times\frac{8}{5}$$
$$=\frac{2}{5}$$

And so, $1\frac{1}{2}+\left(\frac{1}{4}\times 1\frac{3}{5}\right)=1\frac{1}{2}+\frac{2}{5}$

$$1\frac{1}{2}+\frac{2}{5}$$
$$1\frac{5(1)+2(2)}{10}=1\frac{9}{10}\ \text{(exact value)}$$

(b) (i) **Required to calculate:** The exact value of $(0.4)^2\times 3.7$

Calculation:

$$(0.4)^2=0.4\times 0.4$$
$$=0.16$$

Hence $(0.4)^2\times 3.7=0.16\times 3.7$

$$\begin{array}{r} 0.16\ \times \\ 3.7 \\ \hline 48 \\ 112 \\ \hline .592 \\ \hline \end{array}$$

$\therefore (0.4)^2\times 3.7=0.592$ (exact value)

(ii) **Required to express:** 0.167 in standard form.
Solution:

$$0\,.\,1\,6\,7$$

Move the decimal point 1 place to the right, which is equivalent to dividing by 10.
That is, $0.167 = 1.67 \div 10$ and is hence 1.67×10^{-1} when expressed in standard form.

2. (a) **Data:** Heights of two toddlers are in the ratio $7:9$.
The height of the shorter toddler = 63 cm.
Required to calculate: The height of the taller toddler
Calculation:
Let the height of the taller toddler be h cm.
Then,

$$7:9 = 63:h$$

$$\frac{7}{9} = \frac{63}{h}$$

$$\therefore 7 \times h = 63 \times 9$$

$$\therefore h = \frac{63 \times 9}{7}$$

$$= 81$$

$\therefore$ Height of the taller toddler = 81 cm.

(b) **Data:** Ben's salary is \$600 basic wage plus 5% commission on sales over \$1000.
Sales in a particular week was \$3160.
Required to calculate: Ben's total earnings for that week
Calculation:
Ben's total earnings is (Basic wage + Commission)
Commission is 5% on sales over \$1000.

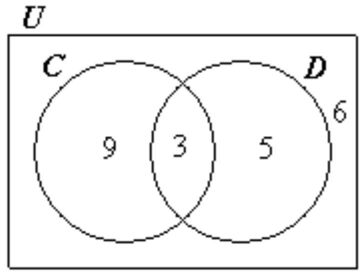
$$\therefore \text{Commission is 5\% of } (\$3160 - \$1000)$$

$$= \frac{5}{100} \times \$2160$$

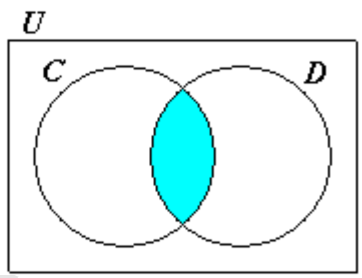
$$= \$108$$

$\therefore$ Ben's total earnings for that week was $\$600 + \$108 = \$708$

3. **Data:** Venn diagram illustrating students who own cats and dogs as pets.
$C = \{\text{Students owning cats}\}$
$D = \{\text{Students owning dogs}\}$



(a) (i) **Required to find:** The number of students who own both cats and dogs.
Solution:
This is shown shaded on the diagram by the region or subset where C and D intersect.



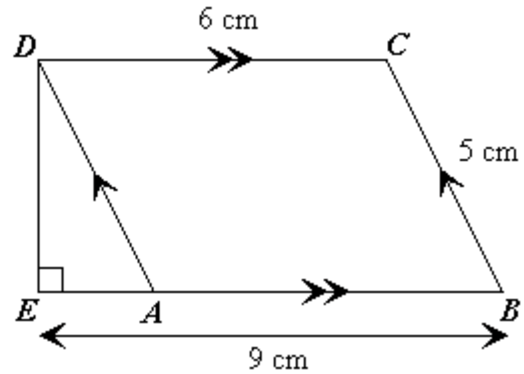
The number of students who own both cats and dogs is 3.

(ii) **Required to describe:** $C' \cap D$
Solution:
C' is the set of elements in U but not in C, that is,
$C' = \{\text{Students who do not own cats}\}$

$\therefore C' \cap D = \{\text{Students who do not own cats and own dogs}\}$

$\therefore C' \cap D$ is the set of students who own only dogs.

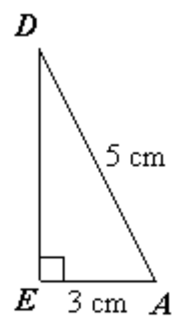
(b) **Data:** Diagram with dimensions as shown.



(i) **Required to calculate:** The length of *DE*.
Calculation:
Length of $EA =$ Length of $EB -$ Length of AB
AB = 6 cm (opposite sides of parallelogram *ABCD* are equal in length)

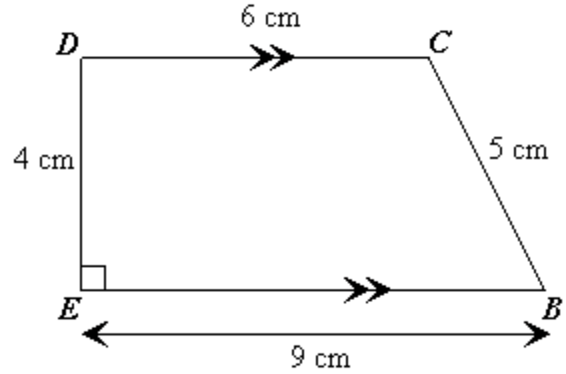
$\therefore EA = 9 - 6$
$= 3$ cm
$AD = BC$
$= 5$ cm (opposite sides of parallelogram *ABCD* are equal in length)

Consider the right angled triangle *AED*.



$ED^2 + (3)^2 = (5)^2$ (Pythagoras' Theorem)
$\therefore ED^2 = (5)^2 - (3)^2$
$= 25 - 9$
$= 16$
$\therefore ED = \sqrt{16}$
$= 4$ cm

(ii) **Required to calculate:** Area of trapezium *BCDE*.
Calculation:



Area of trapezium
$$=\frac{1}{2}(\text{Sum of parallel sides})\times\text{Perpendicular distance between them}$$

$$\text{Area of } BCDE=\frac{1}{2}(6+9)\times 4$$
$$=30\text{ cm}^2$$

4. (a) **Data:** Mr. Singh travels at 80 kmh^{-1} for a distance of *d* km.

(i) **Required to find:** An expression, in *d*, for the time taken.
Solution:

$$\text{Time taken}=\frac{\text{Total distance covered}}{\text{Average speed}}$$
$$=\frac{d\text{ km}}{80\text{ kmh}^{-1}}$$
$$=\frac{d}{80}\text{ hours}$$

(ii) **Data:** Time taken = 30 minutes
Required to calculate: *d*
Calculation:
Time $=30$ minutes
$$=\frac{30}{60}\text{ hours}$$
$$=\frac{1}{2}\text{ hour}$$

$$\therefore \frac{d}{80} = \frac{1}{2}$$
$$2 \times d = 80 \times 1$$
$$\therefore d = \frac{80}{2} \text{ km}$$
$$= 40 \text{ km}$$

(b) **Data:** $C = \frac{5}{9}(F - 32)$, C = temperature in °C and F = temperature in ° Fahrenheit.
Required to convert: 95°F to °C.
Solution:
$$F = 95$$
$$C = \frac{5}{9}(95 - 32)$$
$$= \frac{5}{9}(63)$$
$$= \frac{5 \times 63}{9} \text{°C}$$
$$= 35\text{°C}$$

Hence, 95°F is equivalent to 35°C.

5. (a) **Data:** A relation maps input and output elements in the table shown.

INPUT x	**OUTPUT** y
1	2
2	5
4	$\square$
5	26

(i) **Required to complete:** The table given.
Solution:
To complete the table, we should try to derive the relationship that connects y and x.

Notice when $x = \text{odd}$, $y = \text{even}$
Notice when x = even, y = odd
The y value appears to be a term in x added to 1
Notice y is larger than x. Hence, x is increased by either multiplying by a number or adding a number.

Observation:

When $y=2$ and $x=1$
$2=(1)^2+1$

When $y=5$ and $x=2$
$5=(2)^2+1$

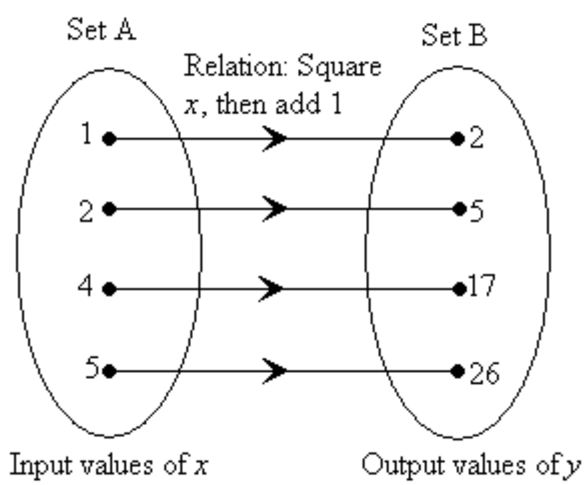
When $y=26$ and $x=5$
$26=(5)^2+1$

$\therefore$ When $x=4$
$y=(4)^2+1$
$=17$

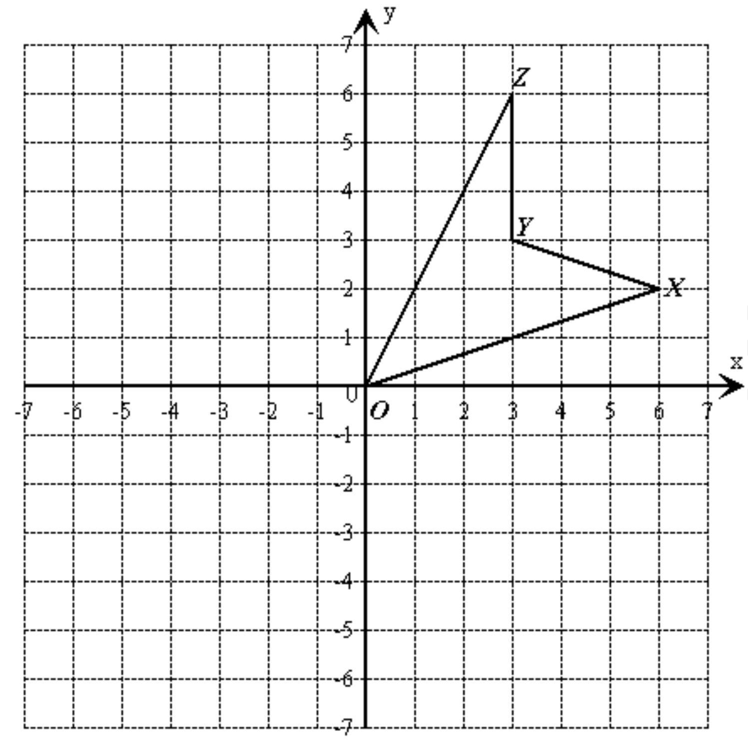
$\therefore$ The missing element in $\boxed{}$ is 17.

(ii) **Required to find:** The relation that connects x and y.
Solution:
The relation that connects x and y is $y=x^2+1$.

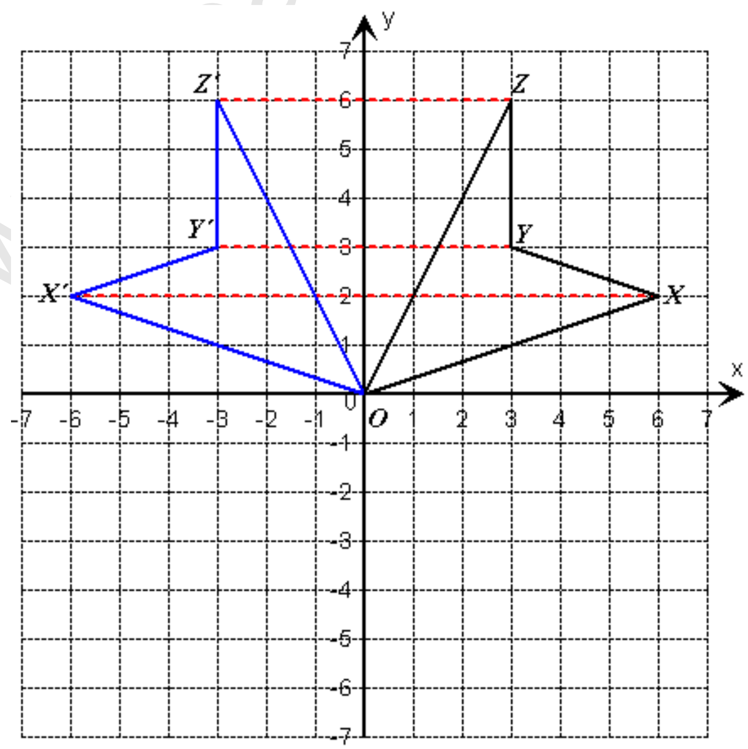
(b) **Required to:** Draw and label an arrow diagram to represent the above relation.
Solution:



6. **Data:** The diagram below shows the plane shape $OXYZ$ with coordinates, $O\left(0,\ 0\right)$, $X\left(6,\ 2\right)$, $Y\left(3,\ 3\right)$ and $Z\left(3,\ 6\right)$.



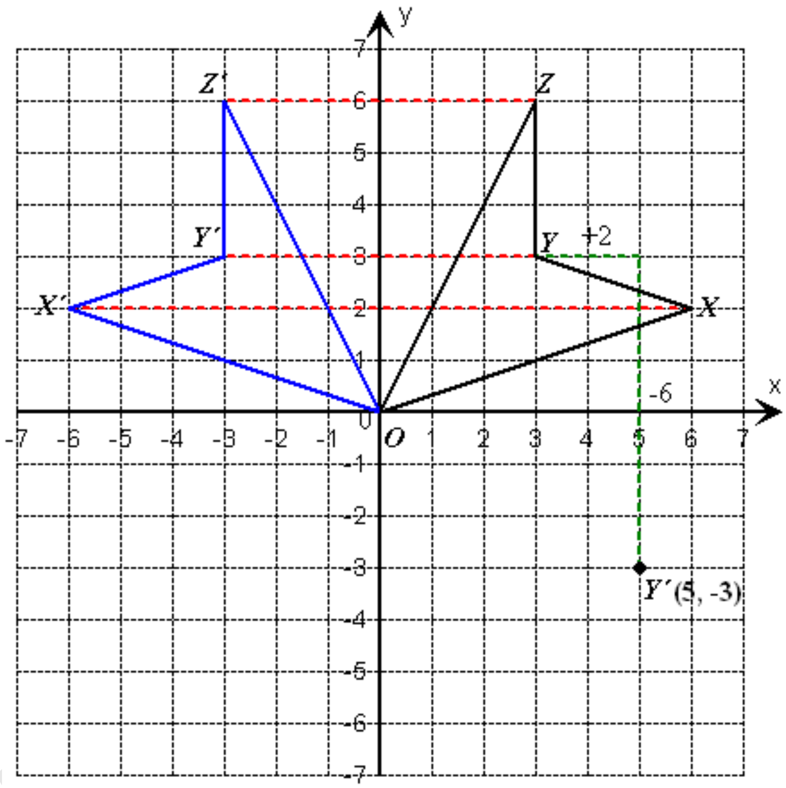
(a) **Required to draw:** The image $O'X'Y'Z'$ of $OXYZ$ after a reflection in the y – axis.
Solution:



$O' = (0, 0)$ (is an invariant point)
$X' = (-6, 2)$
$Y' = (-3, 3)$
$Z' = (-3, 6)$

(b) **Data:** Y is translated to Y' by 2 units parallel to the x – axis and – 6 units parallel to the y – axis.

(i) **Required to locate:** Y'
Solution:



(ii) **Required to state:** The coordinates of Y'.
Solution:
By drawing, we can read off the coordinates of Y' as $(5, -3)$.

OR

$$Y \xrightarrow{T=\begin{pmatrix}2\\-6\end{pmatrix}} Y'$$

Coordinates of Y are $(3,\ 3)$.

$$\therefore \begin{pmatrix}3\\3\end{pmatrix} \xrightarrow{T=\begin{pmatrix}2\\-6\end{pmatrix}} \begin{pmatrix}3+2\\3+-6\end{pmatrix}=\begin{pmatrix}5\\-3\end{pmatrix}$$

$\therefore Y'=(5,\ -3)$

Section II

7. (a) (i) **Data:** Cost to produce 1 chicken pie = \$1.25
Required to calculate: The cost to produce 20 chicken pies
Calculation:
Cost to produce 1 chicken pie = \$1.25

$\therefore$ Cost to produce 20 chicken pies $= \$1.25 \times 20$
$= \$25.00$

(ii) **Data:** Selling price of 1 chicken pie = \$3.00
Required to calculate: Profit on selling all 20 chicken pies.
Calculation:
The total selling price on all 20 chicken pies at \$3.00 each $= \$3.00 \times 20$
$= \$60.00$

Total profit = Total amount received on sales – Total cost of production
$= \$60.00 - \$25.00$
$= \$35.00$

(b) **Data:** Cafeteria produces fruit punch at \$2.50 per bottle and sells at \$4.00 per bottle. Cafeteria produces 20 bottles and sells 14 bottles.
Required to calculate:
Profit or loss on the production of fruit punch
Calculation:
Total cost of production of 20 bottles of fruit punch at \$2.50 each $= \$2.50 \times 20$
$= \$50.00$

Total earned on selling 14 bottles of fruit at \$4.00 each $= \$4.00 \times 14$
$= \$56.00$

Amount earned on sales > Total spent on production.
$\therefore$ A profit is realised.

Profit = Total earned on sales – Total spent on production
$= \$56.00 - \$50.00$
$= \$6.00$

(c) **Data:** Scores obtained by 30 students in a spelling test.

1	0	4	5	4	4	3	1	5	4	3	2	2	1	3
5	5	3	4	3	4	3	3	3	0	2	3	2	2	2

(i) **Required to complete:** The following table.

Test Score	Frequency
0	2
1	
2	
3	
4	
5	

Solution:

By checking the set of values given in the raw data, we obtain:

Test Score, x	Frequency, f
0	2
1	3
2	6
3	9
4	6
5	4
	$\sum f = 30$

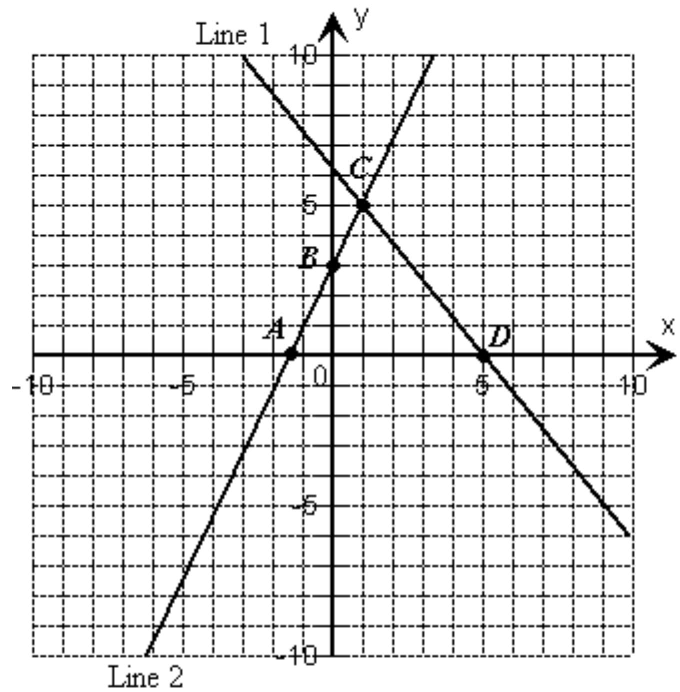
(ii) **Required to find:** The number of students who spelt 2 words correctly.
Solution:
From the table, the number of students who spelt exactly two words correctly is 6.

(iii) **Required to calculate:** The number of students who spelt 3 or more words correctly.
Calculation:
The number of students who spelt 3 or more words correctly
= Number who spelt 3 words correctly
+ Number who spelt 4 words correctly
+ Number who spelt 5 words correctly

$= 9 + 6 + 4$

$= 19$

8. **Data:** The diagram below shows the graphs of two linear functions.



(a) **Required to state:** The coordinates of B
Solution:
The line 2 cuts the y-axis at B and at the point 3.
$\therefore B$ has coordinates $(0, 3)$.

(b) **Required to state:** The coordinates of the point of intersection of the two linear graphs.
Solution:
The two linear graphs intersect at C. Therefore C has coordinates $(1, 5)$.

(c) **Data:** The equation of line 1 is suggested to be $y = 2x + 3$ by Sham and $y = -x + 6$ by Brian.
Required to determine: Whether Sham or Brian is correct
Solution:
By observation, line 1 has a negative gradient and Sham's suggestion of equation $y = 2x + 3$ has a positive gradient of 2. (Expressed in the form $y = mx + c$, where $m = 2 =$ gradient)
$\therefore$ Sham is incorrect and we deduce, by elimination, that Brian is correct.

OR

Determining the equation of line 1:
Using $C=(1,5)$ and $D=(6,0)$ to determine the gradient of line 1.

$$\text{Gradient } = \frac{0-5}{6-1}$$
$$= \frac{-5}{5}$$
$$= -1$$

Line 1 cuts the y – axis at 6.
$\therefore$ Equation of line 1 in the form $y=mx+c$ is $y=-1x+6$, which is, $y=-x+6$, where $m=-1$ and $c=6$.
$\therefore$ Brian is correct.

OR

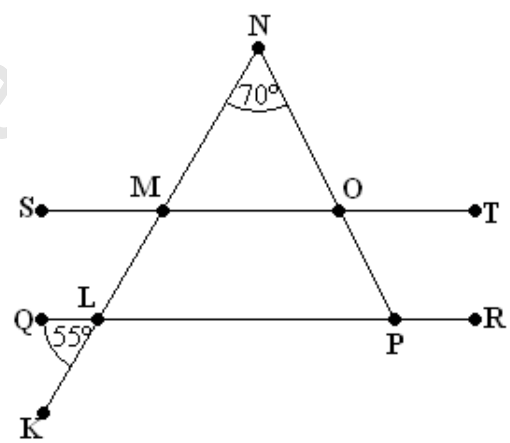
We could have chosen a point, say $(6, 0)$ and use $\frac{y-0}{x-6}=-1$ to obtain the equation of the line.

$y-0=-1(x-6)$

$y=-x+6$

We see that Brian is correct

(d) **Data:** In the diagram below, the line QR is parallel to the line ST and angle KLQ = 55° and $\text{M}\hat{\text{N}}\text{O}=70°$.



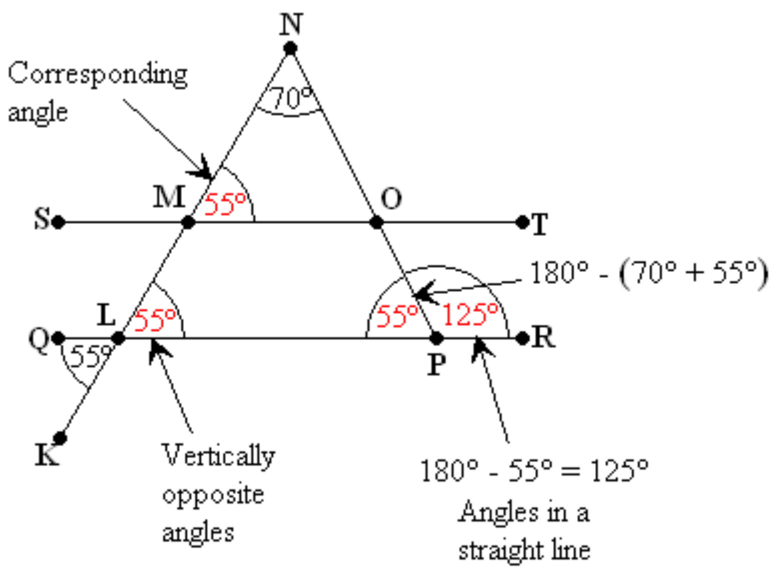
(i) **Required to calculate:** $\text{M}\hat{\text{L}}\text{P}$
Calculation:

$\text{M}\hat{\text{L}}\text{P} = \text{K}\hat{\text{L}}\text{Q}$
$= 55°$
(Vertically opposite angles)

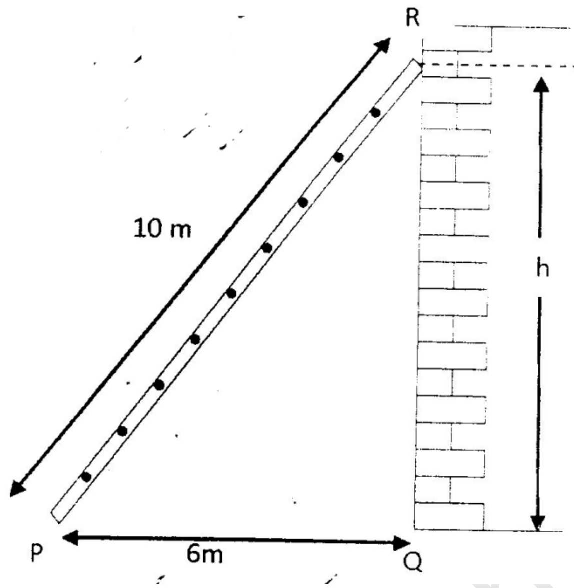
(ii) **Required to calculate:** $N\hat{M}O$
Calculation:
$N\hat{M}O = 55°$
(Corresponding angles to $M\hat{L}P$)

(iii) **Required to calculate:** $O\hat{P}R$
Calculation:
Consider ΔNLP
$N\hat{P}L = 180° - (70° + 55°)$
$= 55°$
(Sum of angles in a triangle = 180°)

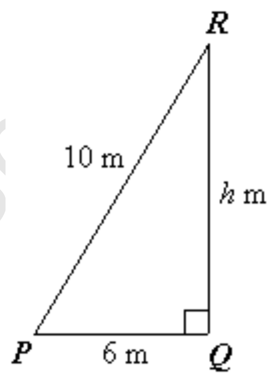
$N\hat{P}R = O\hat{P}R$ (the same angle)
$= 180° - 55°$
$= 125°$
(Angles in a straight line)



9. (a) (i) **Data:** The diagram shows a ladder leaning against a wall.

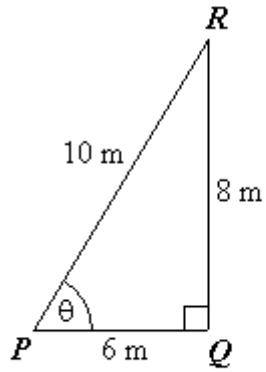


Required to calculate: h
Calculation:
Consider ΔPQR



$$(h)^2 + (6)^2 = (10)^2 \qquad \text{(Pythagoras' Theorem)}$$
$$\therefore h^2 = 100 - 36$$
$$= 64$$
$$\therefore h = \sqrt{64}$$
$$= 8 \text{ m}$$

(ii) **Required to calculate:** The angle that the ladder makes with the ground.
Calculation:
The angle required is $R\hat{P}Q$.



Let $R\hat{P}Q = \theta$

$$\sin\theta = \frac{8}{10}$$
$$\therefore \theta = \sin^{-1}\left(\frac{8}{10}\right)$$
$$= 53.1\underline{\underline{3}}^\circ$$
$$= 53.1^\circ \text{ to the nearest } 0.1^\circ$$

OR

$$\cos\theta = \frac{6}{10}$$
$$\therefore \theta = \cos^{-1}\left(\frac{6}{10}\right)$$
$$= 53.1\underline{\underline{3}}^\circ$$
$$= 53.1^\circ \text{ to the nearest } 0.1^\circ$$

OR

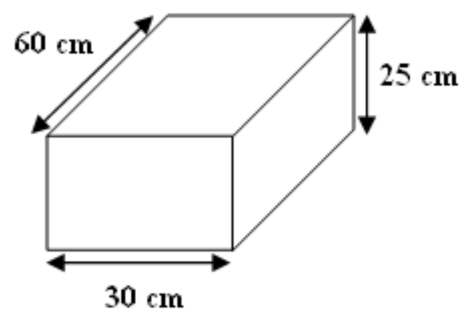
$$\tan\theta = \frac{8}{6}$$

$$\therefore \theta = \tan^{-1}\left(\frac{8}{6}\right)$$

$$= 53.1\underline{\underline{3}}^\circ$$

$$= 53.1^\circ \text{ to the nearest } 0.1^\circ$$

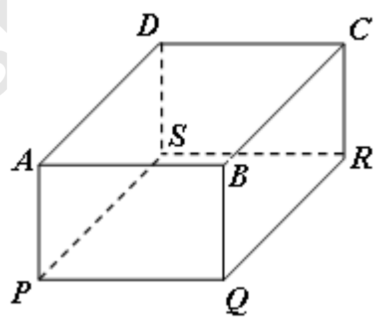
(b) (i) **Data:** The cuboid below represents a closed box with the following dimensions: length = 60 cm, width = 30 cm and height = 25 cm.



Required to Draw: A net of the cuboid clearly showing the dimension of each face

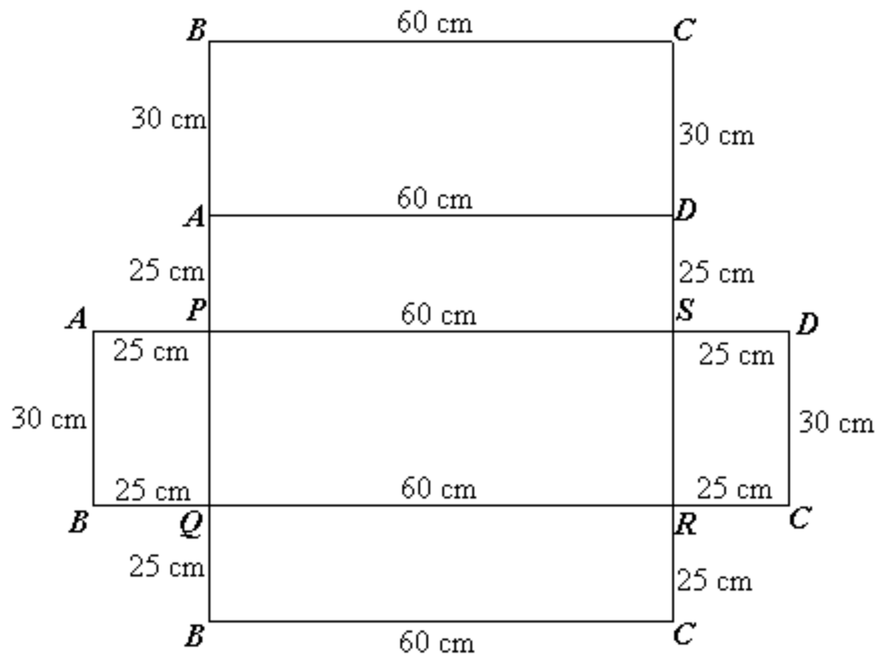
Solution:

First, we name the vertices *A*, *B*, *C*, *D*, *P*, *Q*, *R* and *S* as shown.



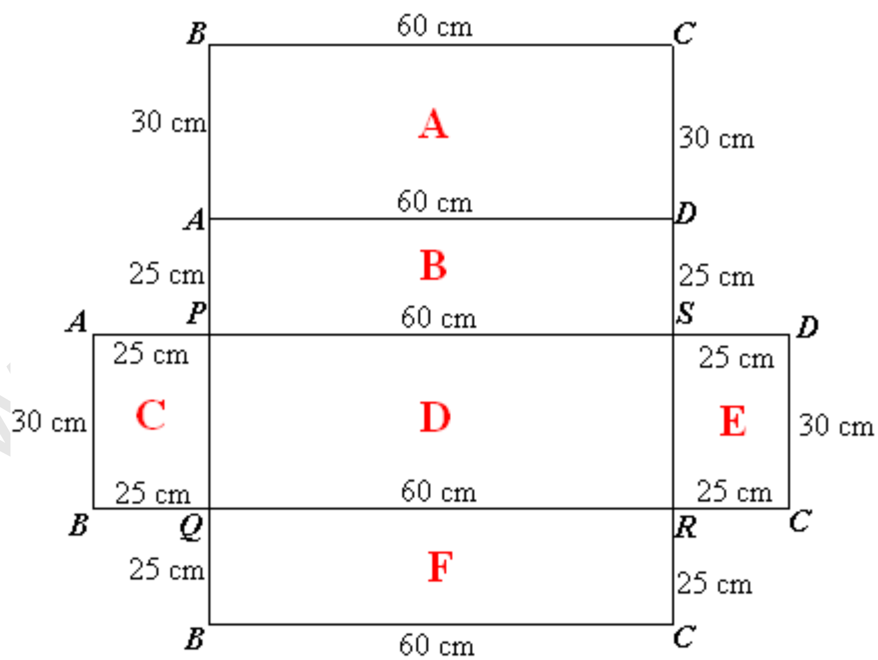
Cutting along the vertical sides *AP*, *BQ, CR* and *DS*, one of the many possible nets can be obtained.

In this case, the net looks like:



(ii) **Required to calculate:** The total surface area of the cuboid, using the net drawn.

Calculation:



Let us name the six faces A – F as shown.

Area of A = Area of D (faces A and D are congruent)

Area of B = Area of F (faces B and F are congruent)

Area of C = Area E (faces C and E are congruent)

Area of A and D $= 2\times\{30\times60\}$
$= 3600 \text{ cm}^2$

Area of B and F $= 2\times\{25\times60\}$
$= 3000 \text{ cm}^2$

Area of C and E $= 2\times\{30\times25\}$
$= 1500 \text{ cm}^2$

$\therefore$ Total surface area of the cuboid is the sum of the areas of all six faces
$= 3600 + 3000 + 1500 \text{ cm}^2$
$= 8100 \text{ cm}^2$