FAS-PASS Maths

CSEC MATHEMATICS MAY-JUNE 2015

SECTION I

1. (a) (i) Using a calculator, or otherwise, determine the EXACT value of:

$$2\frac{2}{5}-1\frac{1}{3}+3\frac{1}{2}$$

SOLUTION:

Required to calculate: The exact value of $2\frac{2}{5}-1\frac{1}{3}+3\frac{1}{2}$.

Calculation:

$$\begin{aligned}2\frac{2}{5}-1\frac{1}{3}+3\frac{1}{2} &= 2\frac{2}{5}+3\frac{1}{2}-1\frac{1}{3}\\ &= 4+\frac{2}{5}+\frac{1}{2}-\frac{1}{3}\\ &= 4+\frac{6(2)+15(1)-10(1)}{30}\\ &= 4+\frac{12+15-10}{30}\\ &= 4+\frac{17}{30}\\ &= 4\frac{17}{30} \text{ (in exact form)}\end{aligned}$$

(ii) Using a calculator, or otherwise, determine the EXACT value of:

$$(4.14 \div 5.75)+(1.62)^2$$

SOLUTION:

Required to calculate: The exact value of $(4.14 \div 5.75)+(1.62)^2$.

Calculation:

$$\begin{aligned}(4.14 \div 5.75)+(1.62)^2 &= (4.14 \div 5.75)+(1.62\times 1.62)\\ &= 0.72+2.6244 \text{ (by the calculator)}\\ &= 3.3444 \text{ (in exact form)}\end{aligned}$$

(iii) Using a calculator, or otherwise, determine the EXACT value of

$$2\times 3.142\times 1.25$$

SOLUTION:

Required to calculate: The exact value of $2 \times 3.142 \times 1.25$.
Calculation:
$2 \times 3.142 \times 1.25 = 7.855$ (in exact form by calculator)

(iv) Using a calculator, or otherwise, determine the EXACT value of

$$\sqrt{2.89} \times \tan 45°$$

SOLUTION:
Required to calculate: The exact value of $\sqrt{2.89} \times \tan 45°$.
Calculation:
Taking the positive root

$$\begin{aligned}\sqrt{2.89} \times \tan 45° &= 1.7 \times 1 \\ &= 1.7 \text{ (in exact form)}\end{aligned}$$

(b) The table below shows a shopping bill for Mrs. Rowe. The prices of some items are missing.

Shopping Bill		
Item	**Unit Cost Price**	**Total Cost Price**
3 kg sugar	**X**	\$10.80
4 kg rice	**Y**	**Z**
2 kg flour	\$1.60	\$3.20

(i) Calculate the value of X, the cost of 1 kg of sugar.

SOLUTION:

Data: Table showing Mrs. Rowe's shopping bill.
Required to calculate: The value of **X**
Calculation:
3 kg of sugar cost \$10.80

$$\begin{aligned}\therefore \text{Cost of 1 kg of sugar } &= \frac{\$10.80}{3} \\ &= \$3.60\end{aligned}$$

$$\therefore \mathbf{X} = \$3.60$$

(ii) If the cost price of 1 kg of rice is 80 cents MORE than for 1 kg of flour, calculate the value of **Y** and of **Z**.

SOLUTION:
Data: 1 kg of rice costs 80¢ more than 1 kg of flour.

Required to calculate: The value of **Y** and of **Z**
Calculation:
1 kg of flour costs \$1.60

$$\therefore \text{the cost of 1 kg of rice } = \$1.60 + \$0.80 \\ = \$2.40$$

$$\therefore \mathbf{Y} = \$2.40$$

$$\text{The cost of 4 kg of rice at \$2.40 per kg } = \$2.40 \times 4 \\ = \$9.60$$

$$\therefore \mathbf{Z} = \$9.60$$

(iii) A tax of 10% of the total cost price of the three items is added to Mrs. Rowe's bill. What is Mrs. Rowe's TOTAL bill, including the tax?

SOLUTION:

Data: 10% of the cost of the items is added on as tax.
Required to calculate: Mrs. Rowe's total bill
Calculation:
Cost of 3 kg of sugar, 4 kg of rice and 2 kg of flour, before tax =

$$\begin{array}{r} \$\,10.80 \\ \$\ \ 9.60\ + \\ \underline{\$\ \ 3.20} \\ \underline{\$23.60} \end{array}$$

$$\text{Tax } = 10\% \text{ of } \$23.60 \\ = \frac{10}{100} \times \$23.60 \\ = \$2.36$$

Hence, Mrs. Rowe's total bill

$$\begin{array}{r} = \$23.60\ + \\ \underline{\$\ \ 2.36} \\ \underline{\$25.96} \end{array}$$

2. (a) Given that $a = 4$, $b = 2$ and $c = -1$, find the value of:

(i) $a - b + c$

SOLUTION:

Data: $a = 4$, $b = 2$ and $c = -1$

Required to calculate: $a-b+c$
Calculation:

$$a-b+c=4-(2)+(-1)$$
$$=1$$

(ii) $2a^b$

SOLUTION:
Required to calculate: $2a^b$
Calculation:

$$2a^b=2(4)^2$$
$$=2\times16$$
$$=32$$

(b) A bottle contains 500 ml of orange juice. Write an expression for EACH of the following. The amount of juice left in the bottle after pouring out

(i) p ml

SOLUTION:
Data: The amount of orange juice in a bottle before pouring is 500 ml.
Required to calculate: The amount of juice after p ml has been poured out
Calculation:
The amount of juice left in the bottle
= The initial amount of juice – The amount of juice poured out
= $(500-p)$ ml

(ii) q glasses each containing r ml.

SOLUTION:

Required to calculate: The amount of juice left after pouring q glasses, each containing r ml

Calculation:
The amount of juice in q glasses with r ml each $=(q\times r)$ ml
$=qr$ ml

$\therefore$ The amount of juice, remaining
= The initial amount before pouring – The amount poured out
$=(500-qr)$ ml

(c) Write as a single fraction, as simply as possible

$$\frac{2k}{3}+\frac{2-k}{5}$$

SOLUTION:

Required to write: $\frac{2k}{3}+\frac{2-k}{5}$ is a single fraction

Solution:

$$\frac{2k}{3}+\frac{2-k}{5}$$

$$\frac{5(2k)+3(2-k)}{15}=\frac{10k+6-3k}{15}$$

$$=\frac{10k-3k+6}{15}$$

$$=\frac{7k+6}{15}$$ (as a single fraction in its simplest form)

(d) Four mangoes and two pears cost \$24.00, while two mangoes and three pears cost \$16.00.

(i) Write a pair of simultaneous equations in x and y to represent the information given above.

SOLUTION:

Data: 4 mangoes and 2 pears cost \$24.00. 2 mangoes and 3 pears costs \$16.00.
Required to write: The information in terms of x and y.
Solution:
***x* and *y* needed to have been defined first and hence part (ii) should have been part (i).**
Let the cost of 1 mango be \$$x$ and the cost of 1 pear be \$$y$.
Hence, the cost of 4 mangoes and 2 pears =

$$(4\times x)+(2\times y)=24$$

$$4x+2y=24$$

$\div 2$

$$2x+y=12 \quad ...(1)$$

And, the cost of 2 mangoes and 3 pears =

$$(2\times x)+(3\times y)=16$$

$$2x+3y=16 \quad ...(2)$$

(ii) State clearly what x and y represent.

SOLUTION:

Required to state: What x and y represent
Solution:
x represented the cost of 1 mango.
y represented the cost of 1 pear.

(e) Factorise completely:

(i) $a^3 - 12a$

SOLUTION:

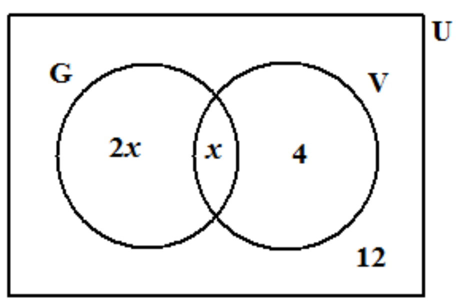
Required to factorise: $a^3 - 12a$
Solution:
$$a^3 - 12a = \underline{\underline{a}} \times a^2 - \underline{\underline{a}} \times 12$$
$$= a\left(a^2 - 12\right)$$

(ii) $2x^2 - 5x + 3$

SOLUTION:

Required to factorise: $2x^2 - 5x + 3$
Solution:
$$2x^2 - 5x + 3 = (2x - 3)(x - 1)$$

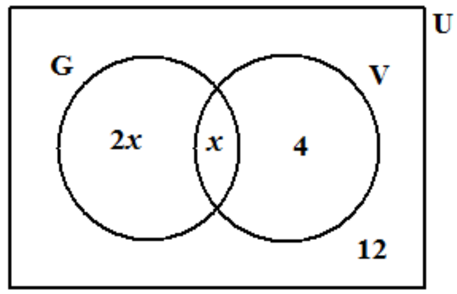
3. (a) The Venn diagram below shows the number of students who play the guitar (G) or the violin (V), in a class of 40 students.



(i) How many students play neither the guitar nor the violin?

SOLUTION:

Data: A Venn diagram showing the number of students who play the guitar (G) or the violin (V) in a class of 40 students.



Required to find: The number of students who do not play either the guitar (G) or the violin (V)
Solution:
The number of students who do not play either the guitar (G) or the violin (V) $= n\left(G \cup V\right)'$
$= 12$ (as illustrated on the diagram)

(ii) Write an expression, in terms of *x*, which represents the TOTAL number of students in the class.

SOLUTION:
Required to write: An expression, in terms of *x*, which represents the total number of students in the class
Solution:
The total number of students in the class is the sum of the numbers of the students in all the subsets of the Universal set $= 2x + x + 4 + 12$
$= 3x + 16$

(iii) Write an equation which may be used to determine the total number of students in the class.

SOLUTION:
Required to write: An equation which may be used to determine the total number of students in the class
Solution:
The equation that may be used to determine the total number of in the class is $3x + 16 = 40$.

(iv) How many students play the guitar?

SOLUTION:
Required to calculate: The number of students who play the guitar
Calculation:

Solving the equation:

$$3x + 16 = 40$$
$$\therefore 3x = 40 - 16$$
$$3x = 24$$
$$x = \frac{24}{3}$$
$$x = 8$$

The number of students who play the guitar $= 2x + x$
$= 3x$

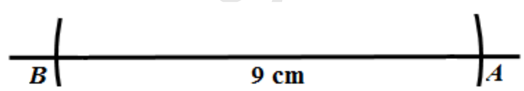
Since $x = 8$, the number of students who play the guitar $= 3(8)$
$= 24$

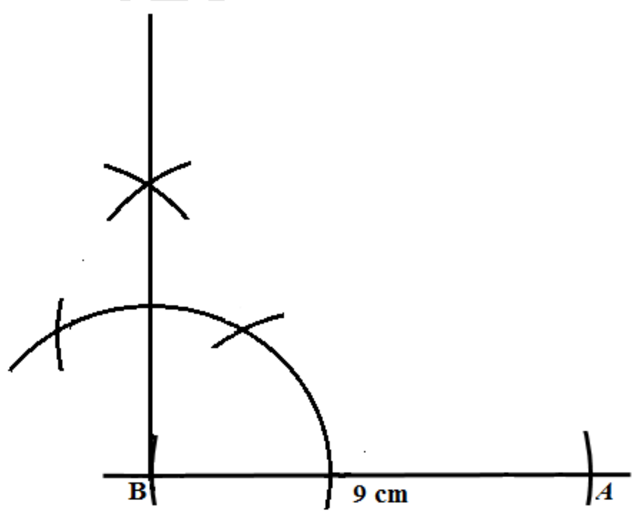
(b) (i) Using **a ruler, a pencil and a pair of compasses**, construct triangle ABC with $AB = 9$ cm, angle $ABC = 90°$ and $BC = 6$ cm.

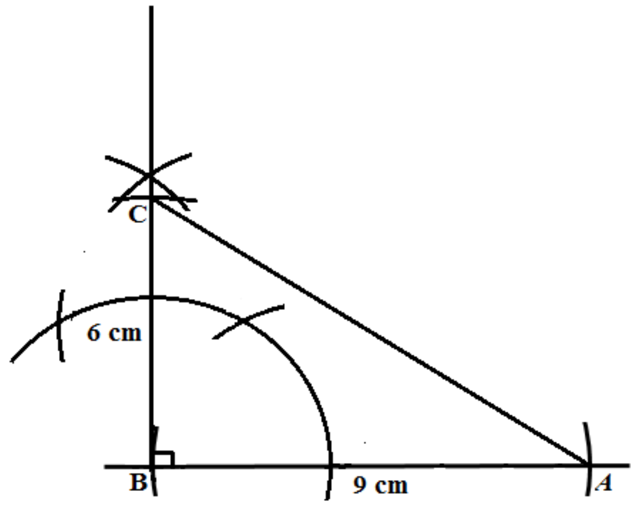
SOLUTION:

Required to construct: Triangle ABC with $AB = 9$ cm, angle $ABC = 90°$ and $BC = 6$ cm.

Construction:







(ii) Measure and state the size of angle *BAC*.

SOLUTION:

Required to measure: And state the size of angle *BAC*
Solution:
Angle $BAC = 34°$ (by measurement)

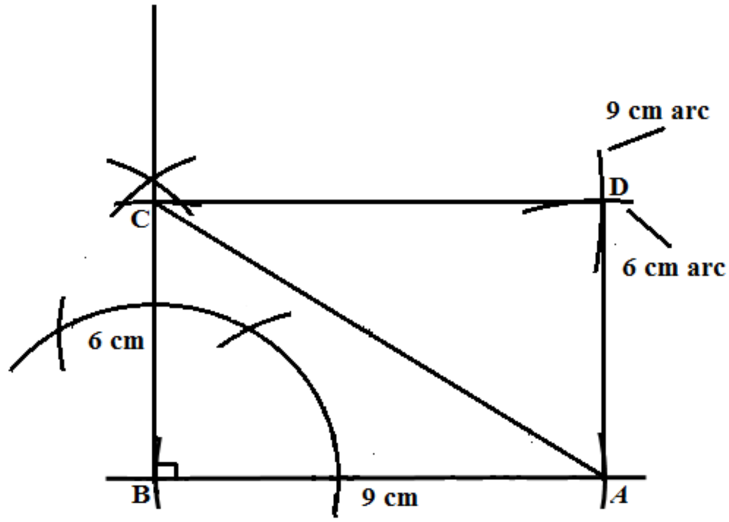
(iii) On the diagram, show the point *D* such that *ABCD* is a parallelogram.

SOLUTION:

Required to show: The point *D* such that *ABCD* is a parallelogram

Solution:
Recall, the opposite sides of a parallelogram are equal and parallel.

With center *A*, an arc of 6 cm is drawn.
With center *C*, an arc of 9 cm is drawn.
Let the two arcs intersect at *D*.



ABCD is a parallelogram and more precisely a rectangle.

4. The table below is designed to show the values of x and y for the function $y = x^2 - 2x - 3$ for integer values of x from – 2 to 4.

x	–2	–1	0	1	2	3	4
y	5		–3	–4	–3		5

(a) Complete the table for the function, $y = x^2 - 2x - 3$.

SOLUTION:

Required to complete: The table for the function, $y = x^2 - 2x - 3$.

Solution:

When $x = -1$

$$y = (-1)^2 - 2(-1) - 3$$
$$= 1 + 2 - 3$$
$$= 0$$

When $x = 3$

$$y = (3)^2 - 2(3) - 3$$
$$= 9 - 6 - 3$$
$$= 0$$

The completed table is shown:

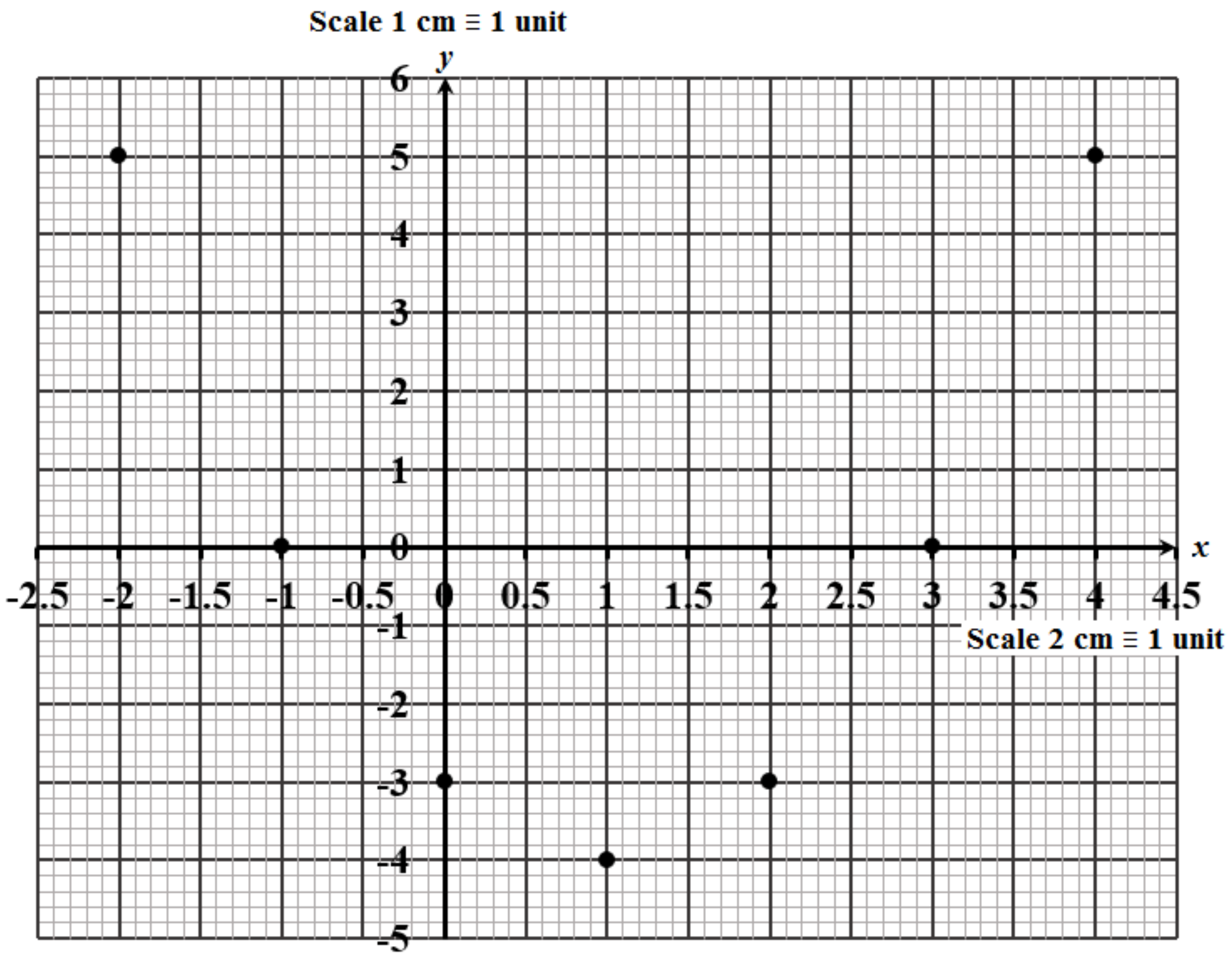
x	–2	–1	0	1	2	3	4
y	5	0	–3	–4	–3	0	5

(b) On the graph, plot the graph of $y = x^2 - 2x - 3$ using a scale of 2 cm to represent 1 unit on the x – axis and 1 cm to represent 1 unit on the y – axis.

SOLUTION:

Required to plot: The points for the graph of $y = x^2 - 2x - 3$

Solution:

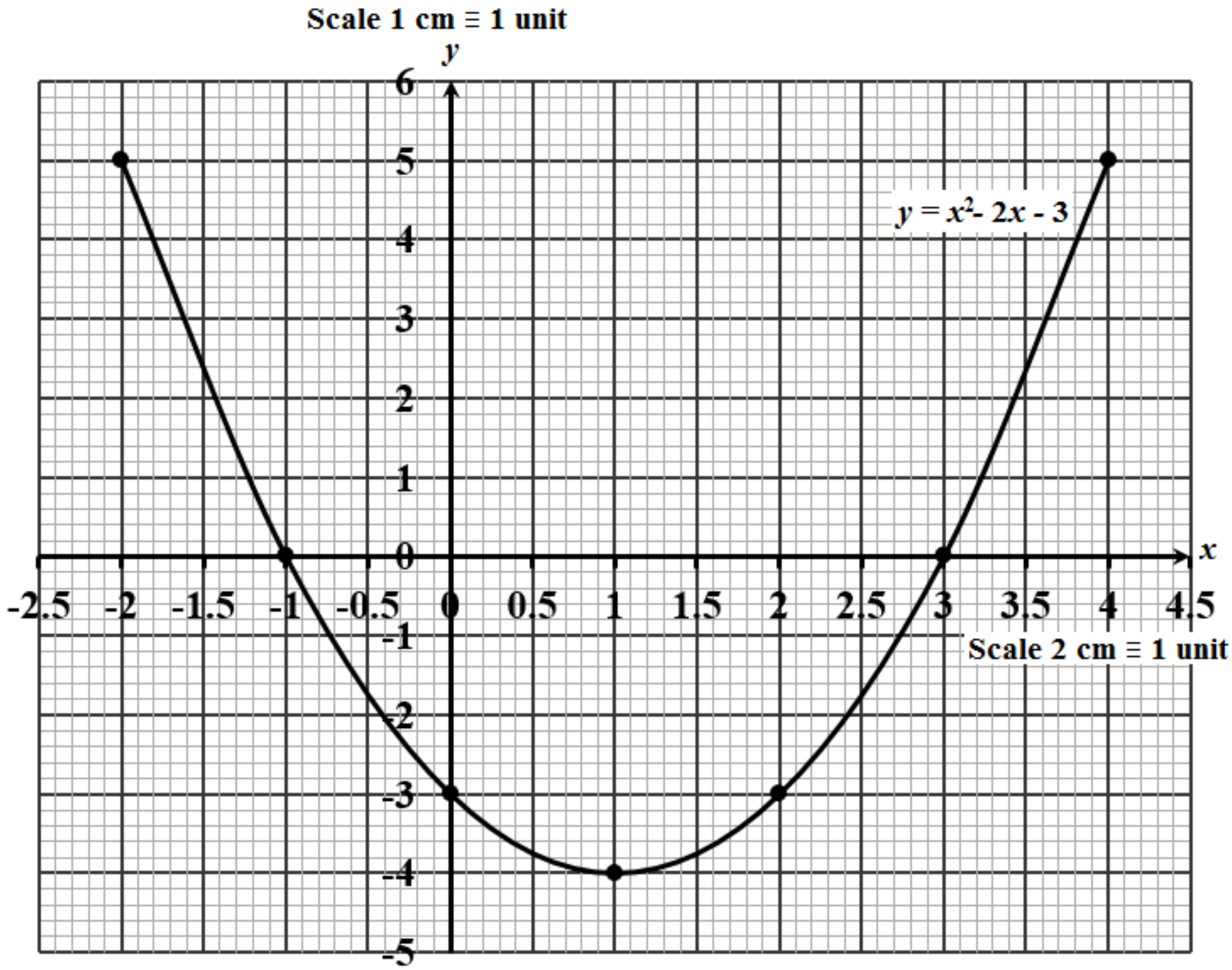


(c) On the graph, draw a smooth curve passing through the points on your graph.

SOLUTION:

Required to draw: A smooth curve passing through the points on the graph

Solution:



(d) Complete the following sentences using information from your graph.

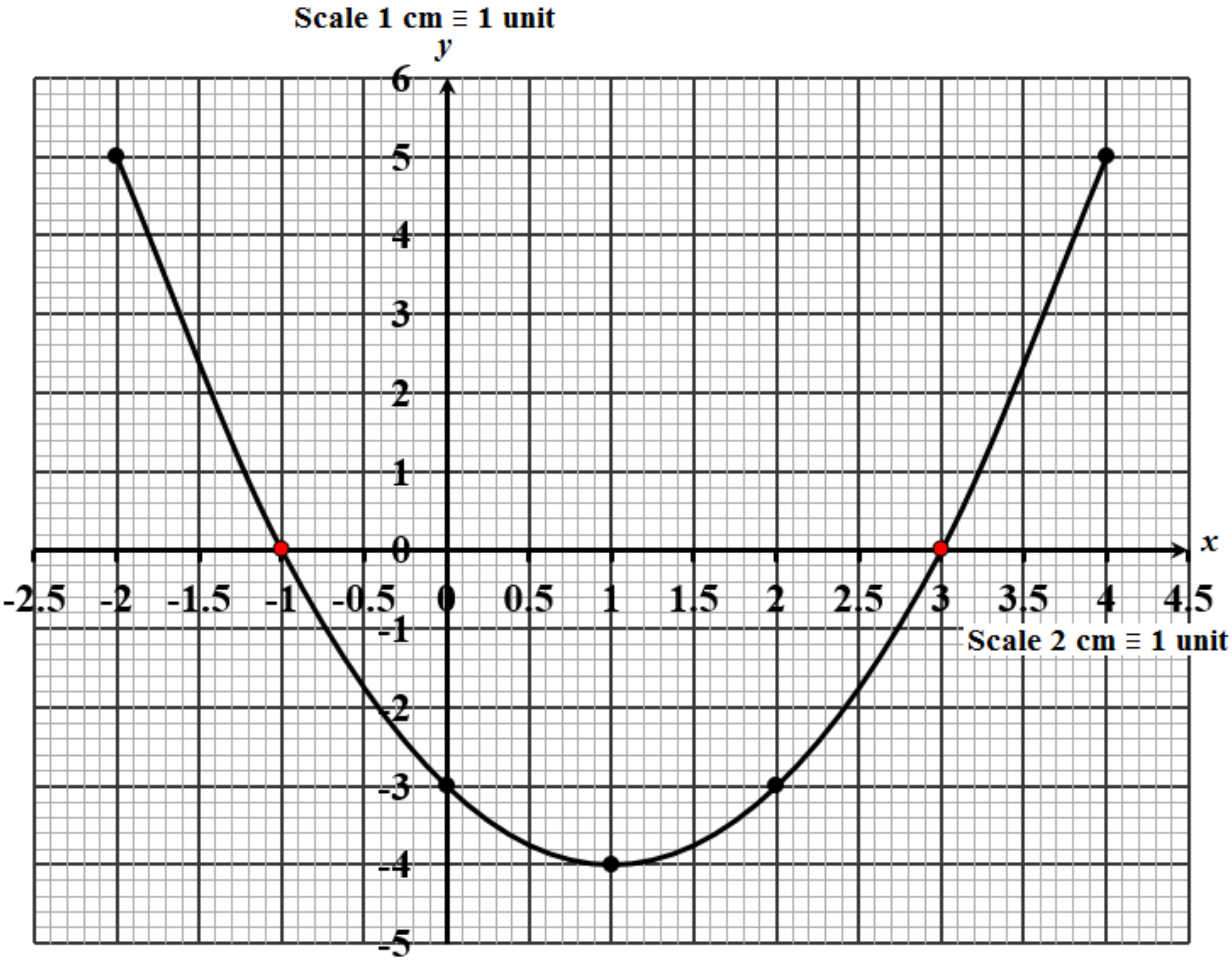
(i) The values of x for which $x^2 - 2x - 3 = 0$ are _______ and _______.

SOLUTION:
Required to complete: The sentence given

Solution:

The values of x for which $x^2 - 2x - 3 = 0$ are the points where the curve cuts the x-axis. These points are shown the diagram.

The values of x for which $x^2 - 2x - 3 = 0$ are $x = -1$ and $x = 3$.

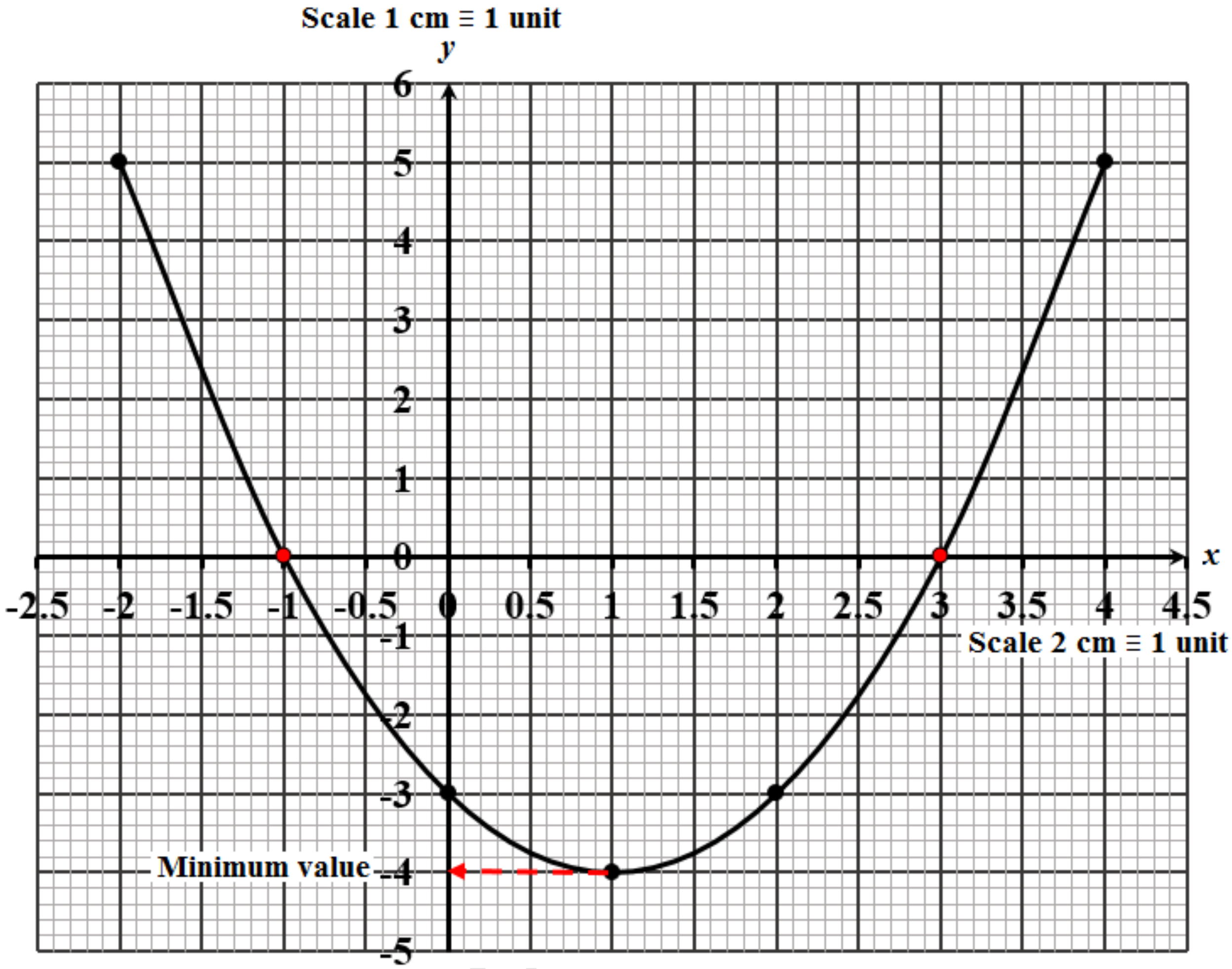


(ii) The **minimum** value of $x^2 - 2x - 3$ is _______.

SOLUTION:
Required to complete: The sentence given.
Solution:
At the minimum point, a horizontal is drawn to meet the vertical axis

The value of y is read off to give the minimum point. This value is shown on the diagram.

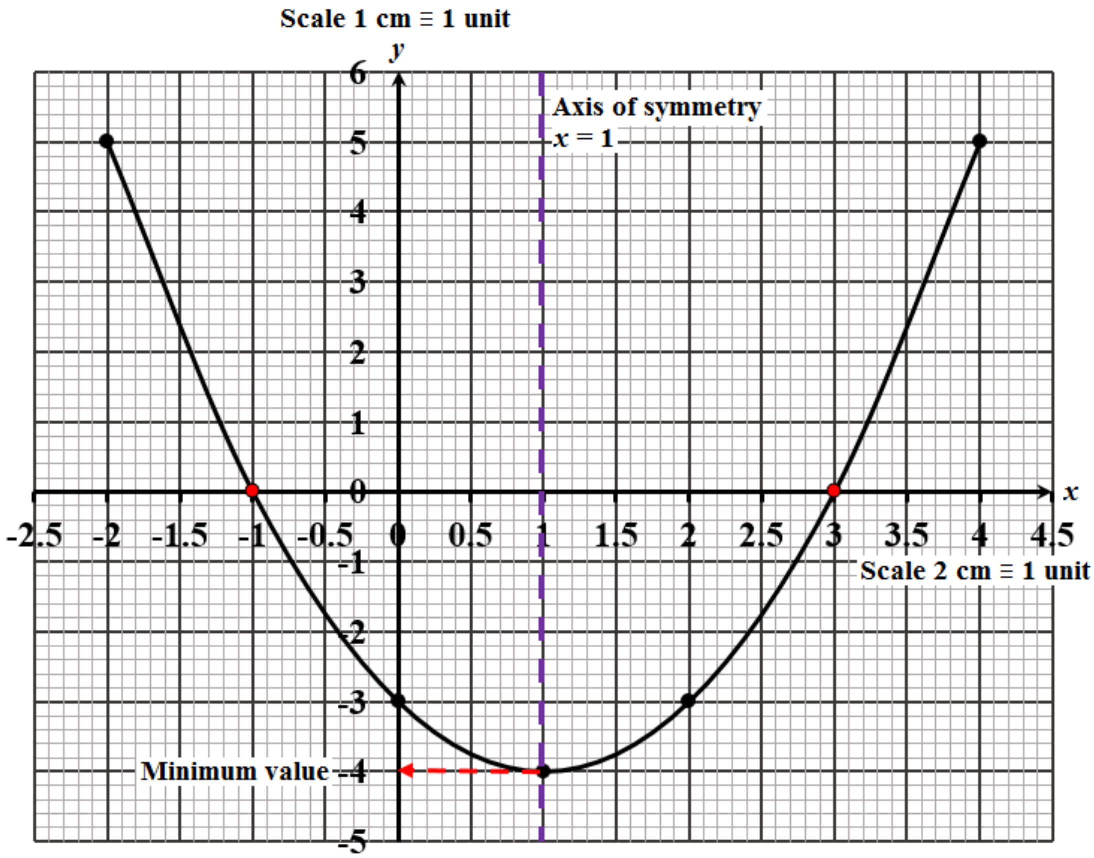
The **minimum** value of $x^2 - 2x - 3$ is -4.



(iii) The equation of the line of symmetry of the graph of $y = x^2 - 2x - 3$ is _______.

SOLUTION:
Required to complete: The sentence given.
Solution:
The line of symmetry is the vertical line that passes through the minimum point of the curve. This line is shown on the diagram.

The equation of the line of symmetry of the graph of $y = x^2 - 2x - 3$ is $x = 1$.



5. (a) A car is travelling at a constant speed of 54 km/h.

(Note-A car cannot travel. It is better to say something like, 'a car was moving at…')

(i) Calculate the distance it travels (covers) in $2\frac{1}{4}$ hours.

SOLUTION:

Data: A car moves at a constant speed of 54 kmh^{-1}.

Required to calculate: The distance covered in $2\frac{1}{4}$ hours

Calculation:

Distance covered in 1 hour = 54 km

$\therefore$ The distance covered in $2\frac{1}{4}$ hours $= 54 \times 2\frac{1}{4}$

$$= \frac{54}{1} \times \frac{9}{4}$$

$$= 121\frac{1}{2} \text{ km}$$

(ii) Calculate the time, in seconds, it takes to travel 315 metres, given that, $1\text{ km/h} = \frac{5}{18}\text{ m/s}$.

SOLUTION:

Data: $1\text{ kmh}^{-1} \equiv \frac{5}{18}\text{ ms}^{-1}$

Required to calculate: The time taken to cover 315 m

Calculation:

$$1\text{ kmh}^{-1} \equiv \frac{5}{18}\text{ ms}^{-1}$$

$$\therefore 54\text{ km}^{-1} \equiv \frac{5}{18} \times 54\text{ ms}^{-1}$$

$$= 15\text{ ms}^{-1}$$

Therefore the time taken to cover 315 m $= \frac{\text{Distance}}{\text{Speed}}$

$$= \frac{315\text{ m}}{15\text{ ms}^{-1}}$$

$$= 21\text{ seconds}$$

(b) Write the following scales in the form $1 : x$.

(i) 1 millimetre = 1 metre

SOLUTION:

Required to write: 1 millimetre = 1 metre in the form $1 : x$.

Solution:

If 1 mm is equivalent to 1 metre, then 1 mm ≡ 1 000 mm

The scale is therefore $1:1000$ and is of the form $1 : x$, where $x = 1000$.

(ii) 2 cm = 6 m

SOLUTION:

Required to write: 2 cm = 6 m in the form $1 : x$.

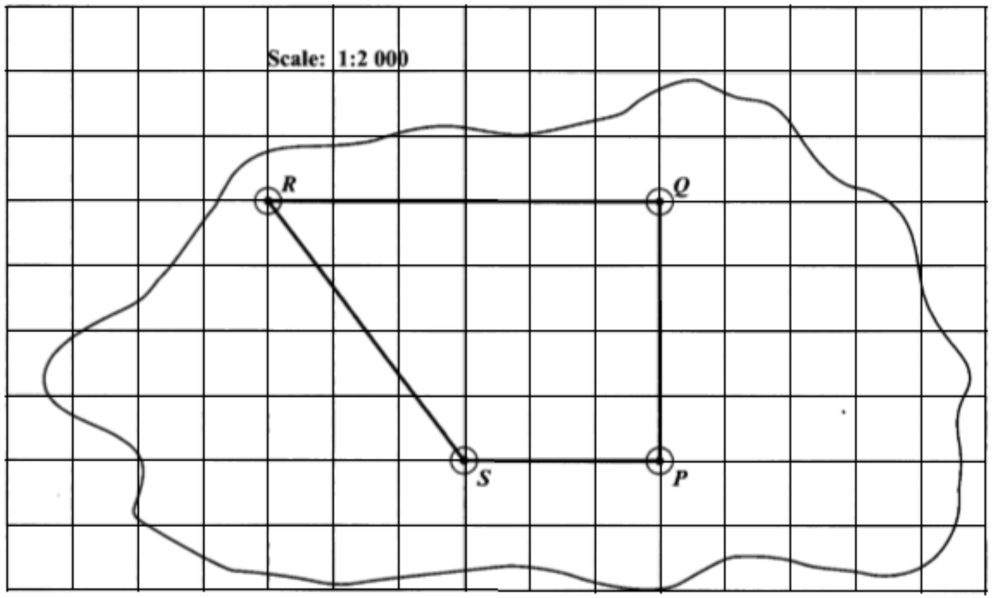
Solution:

Recall 1 metre (m) = 100 centimetres (cm)

If 2 cm is equivalent to 6 m, then 2 cm ≡ 600 cm and 1 cm ≡ 300 cm.

The scale is therefore $1:300$ and is of the form $1 : x$, where $x = 300$.

(c) The map shown below is drawn on a grid of 1 cm squares. P, Q, R and S are four tracking stations. **The scale of the map is** $\mathbf{1:2000}$.



(i) Determine, in centimetres, the distance from Q to R on the map.

SOLUTION:

Data: Map drawn on a grid showing four tracking stations P, Q, R and S. The scale of the map is 1:2 000.
Required to determine: The distance from Q to R
Solution:
The distance from Q to R is '6 blocks' $= 6 \times 1$
$= 6$ cm

(ii) Determine, by counting, the area in square centimetres of the plane $PQRS$ on the map.

SOLUTION:

Required to determine: The area of $PQRS$ by counting

Solution:
Using the system that a region which occupies $\frac{1}{2}$ or more of a square is taken as 1 square and a region that occupies less than $\frac{1}{2}$ of a square is ignored.

Row 1 – 6 squares
Row 2 – 5 squares
Row 3 – 4 squares
Row 4 – 3 squares

Total area of the region $= 6 + 5 + 4 + 3$
$= 18$ squares
$= 18 \times 1\text{cm}^2$
$= 18 \text{ cm}^2$

(iii) Calculate the ACTUAL distance, in kilometres, between Q and R.

SOLUTION:

Required to calculate: The actual distance from Q to R
Calculation:
Distance from Q to R = 6 cm
$\therefore$ Actual distance from Q to R $= 6 \times 2\,000$ cm

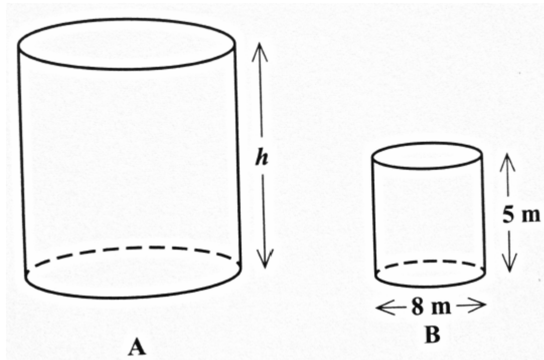
$$= \frac{6 \times 2\,000}{1000 \times 100} \text{ km}$$
$$= \frac{12}{100}$$
$$= 0.12 \text{ or } \frac{3}{25} \text{ km}$$

(iv) Calculate the ACTUAL area, in square metres, of the plane $PQRS$.

SOLUTION:

Required to calculate: The actual area of the plane $PQRS$
Calculation:
The area of $PQRS$ = 18 cm^2
Since 1 cm = 2000 cm, so 1cm^2 = 2000 × 2000 cm^2
$\therefore$ The actual area of $PQRS = 18 \times 2\,000 \times 2\,000 \text{ cm}^2$
Also, 1 m = 100 cm, so 1 m^2 = 100 × 100 cm^2
Actual area of the $PQRS$ in square metres:
$$= \frac{18 \times 2\,000 \times 2\,000}{100 \times 100} \text{ m}^2$$
$$= 18 \times 20 \times 20 \text{ m}^2$$
$$= 7200 \text{ m}^2$$

6. (a) The diagram below, not drawn to scale, shows two cylindrical water tanks, A and B. Tank B has a base diameter 8 m and height 5 m. Both tanks are filled with water.



Take $\pi = 3.14$

(i) Calculate the volume of water in Tank B.

SOLUTION:

Data: Diagram showing 2 cylindrical tanks, A and B.
Required to calculate: The volume of tank B
Calculation:
Volume of tank B $= \pi r^2 h$ (where r = radius and h = vertical height)
Diameter = 8 m
Radius $= \frac{8}{2} = 4$ m

$\therefore$ Volume of tank B $= 3.14 \times (4)^2 \times 5 \text{ m}^3$
$= 251.2 \text{ m}^3$

(ii) If the area of the base of A is 314 m^2, calculate the length of the radius of Tank A.

SOLUTION:

Data: Tank A has a base area of 314 m^2.
Required to calculate: The length of the radius of Tank A
Calculation:
Area of the circular base $= \pi r^2$

$\therefore 3.14 \times r^2 = 314$

$\therefore r^2 = 100$

$r = \sqrt{100}$

$= \pm 10$

$r > 0$

$\therefore$ The radius of Tank A is 10 m.

(iii) Tank A holds 8 times as much water as Tank B. Calculate the height, h, of Tank A.

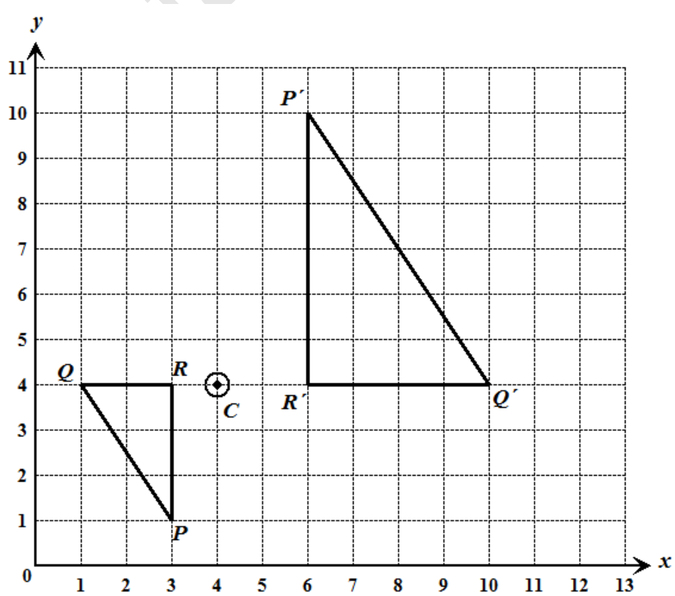
SOLUTION:

Data: Tank A holds 8 times as much water as Tank B
Required to calculate: h
Calculation:
Volume of water in Tank B = 251.2 m^3
$\therefore$ Volume of water in Tank A $= 251.2 \times 8$ m^3

Area of the base of Tank A = 314 m^2

$$\therefore h = \frac{251.2 \times 8}{314} \text{ m}$$
$$= 6.4 \text{ m}$$

(b) The diagram below shows triangle PQR and its image, triangle $P'Q'R'$, after an enlargement centered at the point C on the diagram.



Use the information from the diagram to complete the statements below.

(i) The size of the scale factor is _______.

SOLUTION:

Data: Diagram showing triangle PQR and its image, triangle $P'Q'R'$ after an enlargement centered at the point C.
Required to complete: The statement given
Solution:

$$\frac{Image\ length}{Object\ length} = \text{Scale Factor}$$

$$= \frac{P'R'}{PR}$$

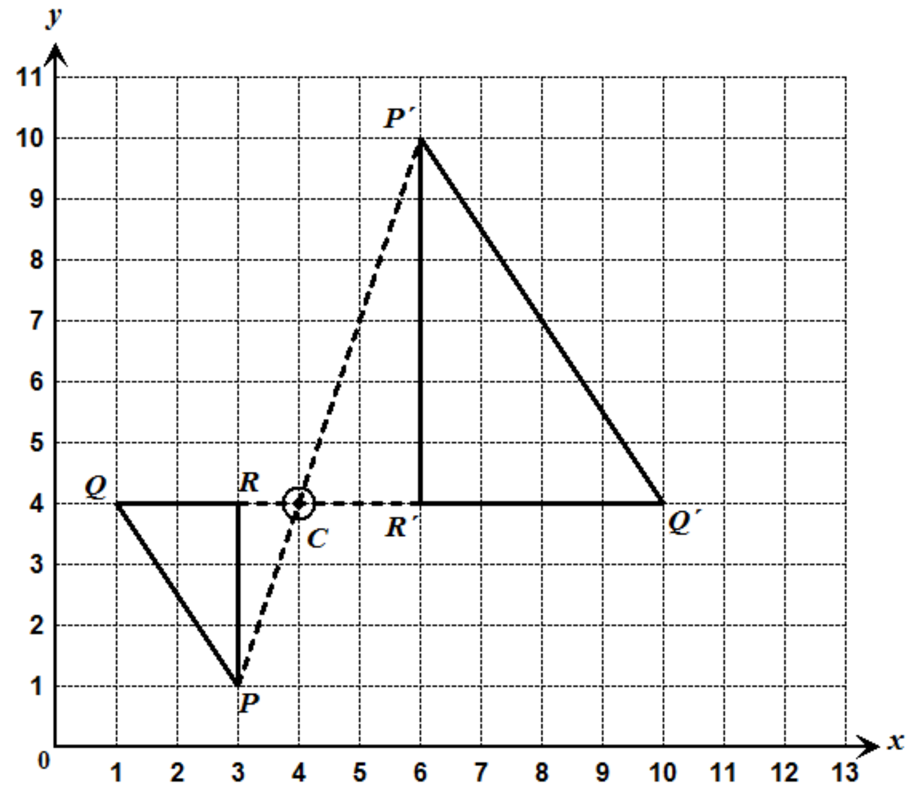
$$= \frac{6}{3} = 2$$

Hence, the scale factor has a magnitude of 2 but the scale factor is – 2.

The size of the scale factor is 2.

(ii) The scale factor is negative because ________________________.

SOLUTION:
Required to complete: The statement given
Solution:
The image is inverted with respect to the object. The center of enlargement lies between the object and the image, that is, the image and the object are on opposite sides of the center of enlargement. This occurs if and only if the scale factor is negative.
ALSO
The scale factor is negative because the image is inverted with respect to the object and the object and image lie on opposite sides of the center of enlargement.



(iii) The length of PQ is $\sqrt{13}$ units, therefore the length of $P'Q'$ is _______.

SOLUTION:

Data: The length of PQ is $\sqrt{13}$ units.
Required to calculate: The length of PQ.
Calculation:
The length of $PQ = \sqrt{13}$ units
Hence, the length of $P'Q' = k \times PQ$, where k is the scale factor

$$= 2\sqrt{13} \text{ units}$$

The length of PQ is $\sqrt{13}$ units, therefore the length of $P'Q'$ is $2\sqrt{13}$ units.

(iv) The area of triangle PQR is ________ square units.

SOLUTION:
Required to calculate: The area of triangle PQR
Calculation:

The area of triangle $PQR = \dfrac{QR \times RP}{2}$

$$= \frac{2 \times 3}{2}$$

$$= 3 \text{ square units}$$

The area of triangle PQR is 3 square units.

(v) The area of $P'Q'R'$ is ________ times the area of triangle PQR which is _______ square units.

SOLUTION:

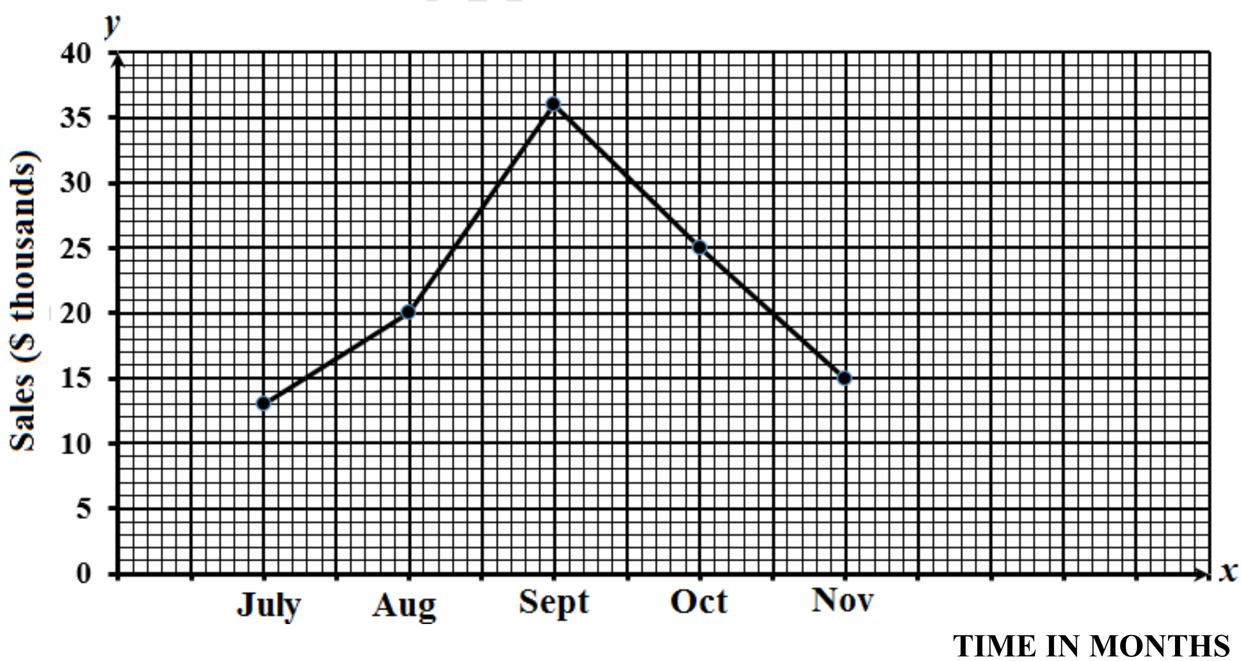
Required to complete: The statement given
Solution:
The area of triangle $P'Q'R' = k^2 \times \text{Area of triangle } PQR$

$$= (2)^2 \times 3 \text{ square units}$$

$$= 12 \text{ square units}$$

The area of $P'Q'R'$ is 4 times the area of triangle PQR which is 12 square units.

7. The line graph below shows the monthly sales, in thousands of dollars, at a car dealership for the period July to November 2014.



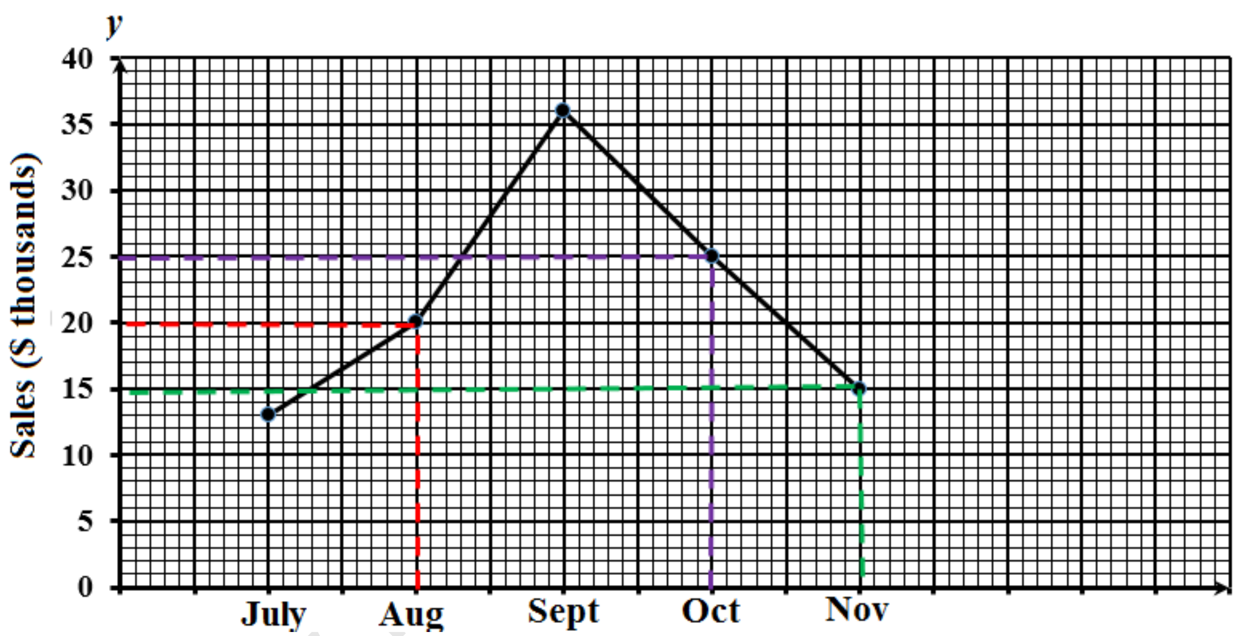
(a) Complete the table below to show the sales for EACH month.

Month	July	August	September	October	November
Sales in $ Thousands	13		36		

SOLUTION:

Data: Line graph showing the monthly sales in thousands of dollars at a car dealership from July to November.
Required to complete: The table given

Solution:



The completed table is:

Month	July	**August**	September	**October**	**November**
Sales in $ Thousands	13	**20**	36	**25**	**15**

(b) (i) Between which TWO consecutive months was there the GREATEST increase in sales?

SOLUTION:

Required to find: The two months between which there was the greatest increase

Solution:
There was an increase in sales between July and August and between August and September only.
Increase in sales from July to August = \$20 000 - \$13 000
= \$7 000

Increase in sales from August to September = \$36 000 - \$20 000
= \$16 000

(Increases occurred between only two periods, so the term 'the greater' increase should be used.)
Hence, **'the greater'** increase in sales was between August and September.

(ii) Between which TWO consecutive months was there the SMALLEST increase in sales?

SOLUTION:
Required to find: The two months between which there was the smallest increase in sales
Solution:

(Increases occurred between only two periods, so the term 'the smaller' increase should be used.)

Hence, the smaller increase in sales occurred between July to August.

(iii) What feature of the line graph enables you to infer that the increase in sales between two consecutive months was the greatest of the smallest?

SOLUTION:
Required to state: The feature of the line graph that indicates the greatest increase in the sales between two consecutive months
Solution:
The gradient of the line over two consecutive months is an indication of the magnitude of the increase. The gradient was larger between August and September than between July and August indicating that there was a larger increase between August and September than between July and August.

(c) Calculate the mean monthly sales for the period July to November 2014.

SOLUTION:

Required to calculate: The mean monthly sales for July to November 2014
Calculation:

The mean monthly sales $= \dfrac{\sum x}{n}$, where x = monthly sales and n = no. of months

$$= \frac{\$13\,000 + \$20\,000 + \$36\,000 + \$25\,000 + \$15\,000}{5}$$

$$= \frac{\$109\,000}{5}$$

$$= \$21800$$

(d) The TOTAL sales for the period July to December was $130 000.

(i) Calculate the sales, in dollars, for the month of December.

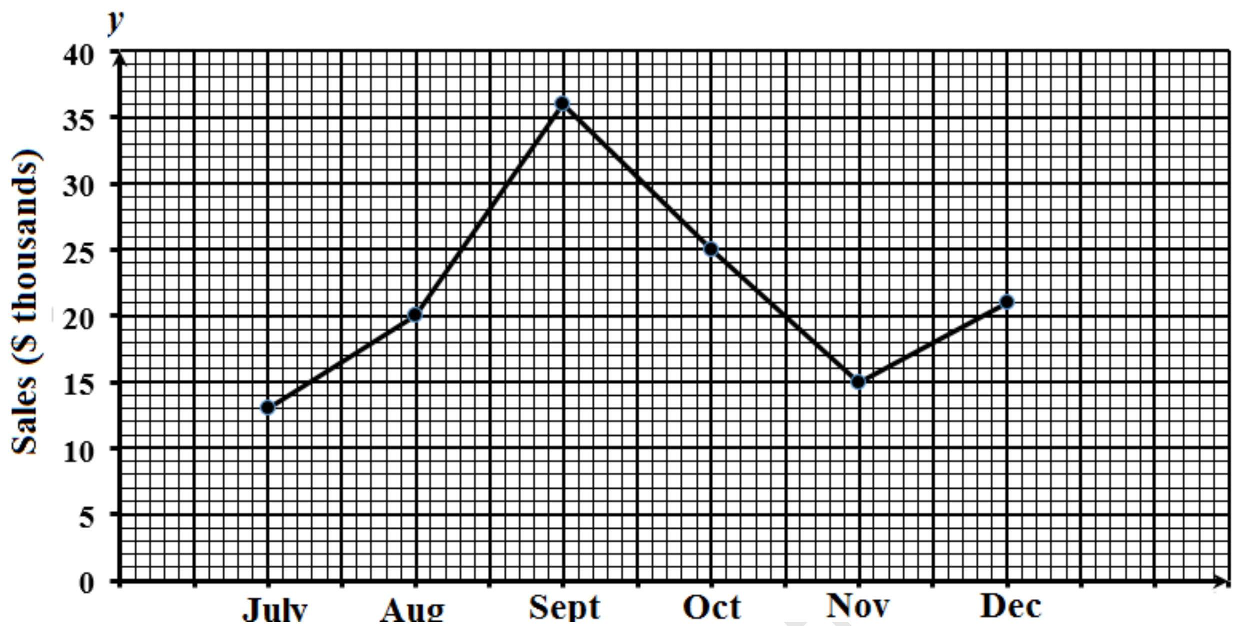
SOLUTION:

Data: The total sales from July to December was $130 000
Required to calculate: The sales in December
Calculation:
Sales from July to November = $109 000
$\therefore$ Sales in December
= Total sales from July to December – Total sales from July to November
$= \$130\,000 - \$109\,000$
$= \$21000$

(ii) Complete the line graph to show the sales for December.

SOLUTION:

Required to complete: The line graph showing the sales for December
Solution:



8. The sequence of figures is made up of equilateral triangles, called unit triangles with unit sides. The first three figures in the sequence are shown below.

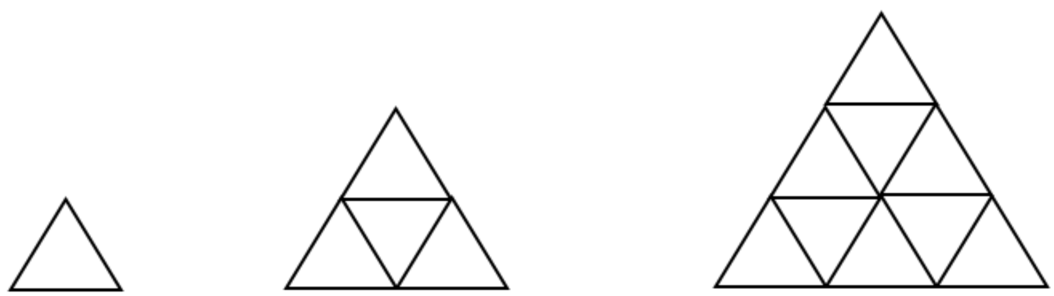
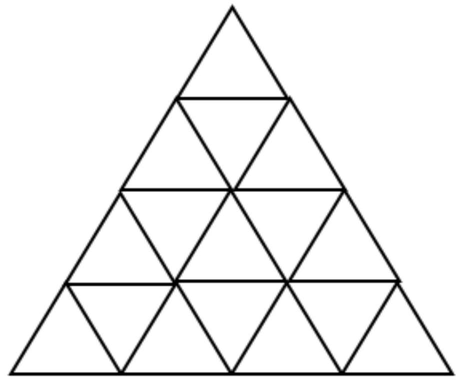


Figure 1 Figure 2 Figure 3

(a) Draw Figure 4 of the sequence.

SOLUTION:

Data: A sequence of 3 figures made of equilateral triangles.
Required to draw: The 4th figure the sequence.
Solution:



(b) Study the patterns of numbers in each row of the table below. Each row relates to one of the figures in the sequence of figures. Some rows have not been included in the table.

Complete the rows numbered (i), (ii), (iii) and (iv).

	Figure	Number of Unit Triangles	Number of Unit Sides
	1	1	$\frac{(3\times1)(1+1)}{2}=3$
	2	4	$\frac{(3\times2)(2+1)}{2}=9$
	3	9	$\frac{(3\times3)(3+1)}{2}=18$
(i)	4		
(ii)		144	
(iii)	25		
(iv)	n		

SOLUTION:

Data: Table showing the relationship between number of unit triangles and number of unit sides in the sequence of diagrams.
Required to complete: The table given
Solution:
By studying the pattern, the following observations are deduced.

Figure	Number of Unit Triangles	Number of Unit Sides
1	1	$\frac{(3\times 1)(1+1)}{2}=3$
2	4	$\frac{(3\times 2)(2+1)}{2}=9$
3	9	$\frac{(3\times 3)(3+1)}{2}=18$
The number of unit triangles is the square of the figure number.		The number of unit sides $=\frac{(3\times \text{Figure number})(\text{Figure number}+1)}{2}$
n	n^2	$\frac{(3\times n)(n+1)}{2}$ $=\frac{3n(n+1)}{2}$

The completed table is:

	Figure	Number of Unit Triangles	Number of Unit Sides
	1	1	$\frac{(3\times 1)(1+1)}{2}=3$
	2	4	$\frac{(3\times 2)(2+1)}{2}=9$
	3	9	$\frac{(3\times 3)(3+1)}{2}=18$
(i)	4	$(4)^2=16$	$\frac{3(4)(4+1)}{2}=30$
(ii)	$\sqrt{144}=12$	144	$\frac{3(12)(12+1)}{2}=234$
(iii)	25	$(25)^2=625$	$\frac{3(25)(25+1)}{2}=975$
(iv)	n	n^2	$\frac{3n(n+1)}{2}$

SECTION II

9. (a) A teacher marks an examination out of 120 marks. The marks are then converted to percentages.

(i) Calculate the percentage for a student who scores

- 60 marks

SOLUTION:

Data: Maximum mark for the examination is 120.
Required to calculate: The percentage for a student who scores 60 marks
Calculation:
Score for student = 60 marks

$$\therefore \text{Percentage score } = \frac{\text{Student's score}}{\text{Maximum score}} \times 100$$
$$= \frac{60}{120} \times 100$$
$$= 50\%$$

- 120 marks

SOLUTION:

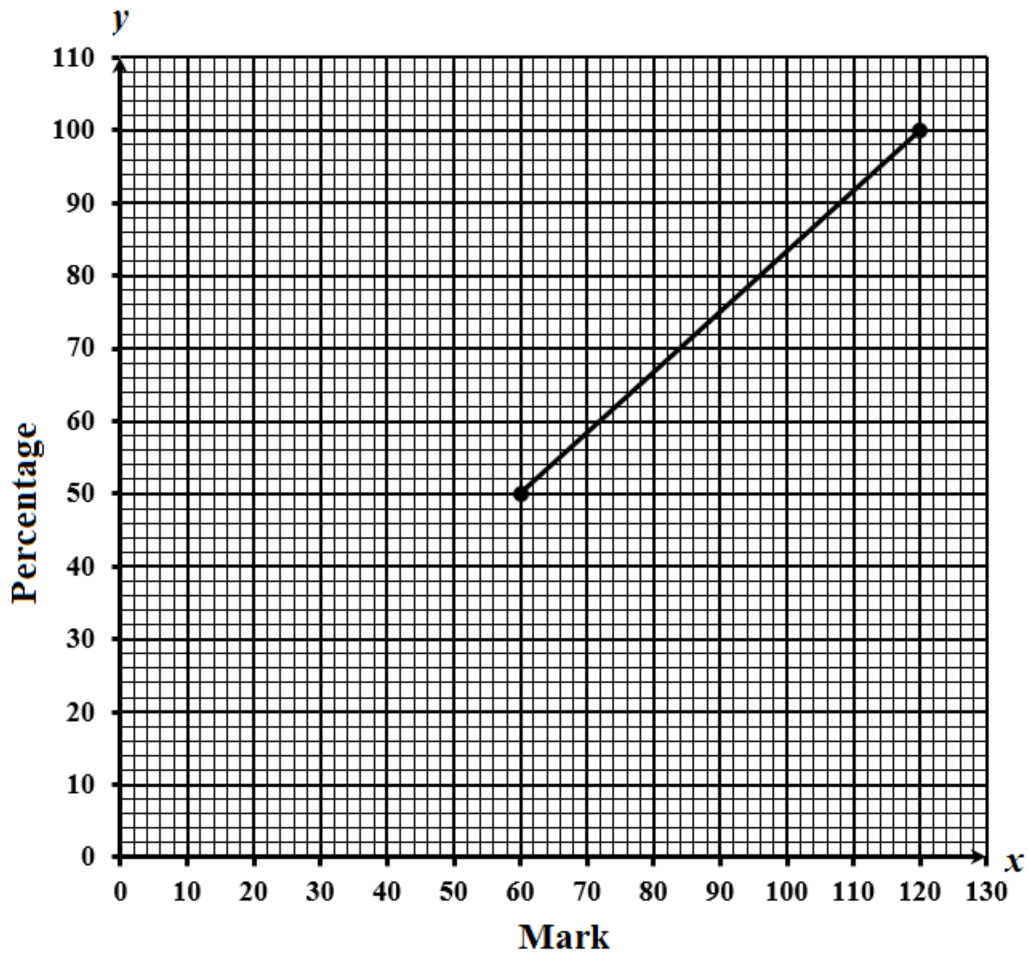
Required to calculate: The percentage for a student who scores 120
Calculation:
Score for student = 120 marks

$$\therefore \text{Percentage score } = \frac{\text{Student's score}}{\text{Maximum score}} \times 100$$
$$= \frac{120}{120} \times 100$$
$$= 100\%$$

(ii) Plot a graph to show the information in (i).

SOLUTION:
Required to plot: Graph to show the information in (i)

Solution:



(iii) A candidate is awarded 95 marks on the examination. Use the graph drawn in (ii) to determine the candidate's percentage.

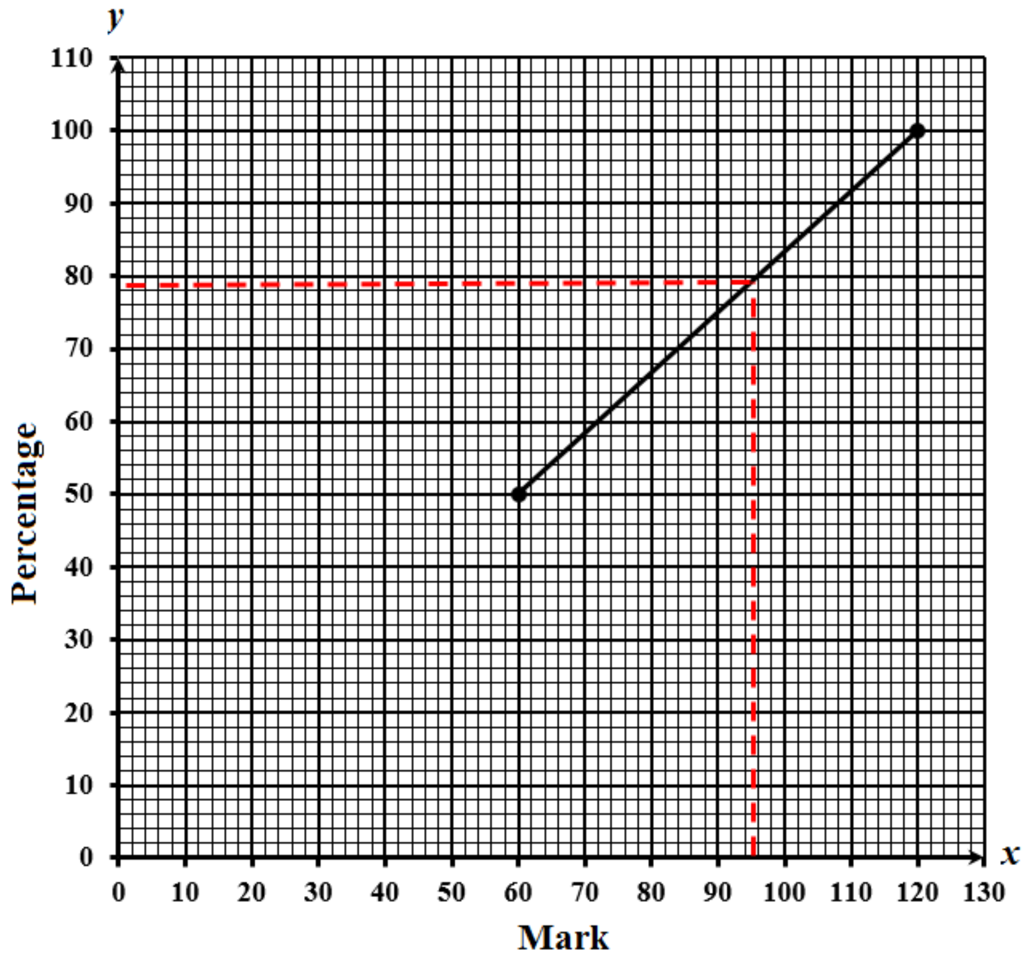
Draw lines on the graph to show how the percentage was obtained.

SOLUTION:

Required to find: A student's percentage if his score was 95, using the graph

Solution:

Student's score is 95 marks. A vertical line is drawn from 95 marks to meet the graph. From that point, a horizontal is drawn to meet the vertical axis. At this point a read off is made.



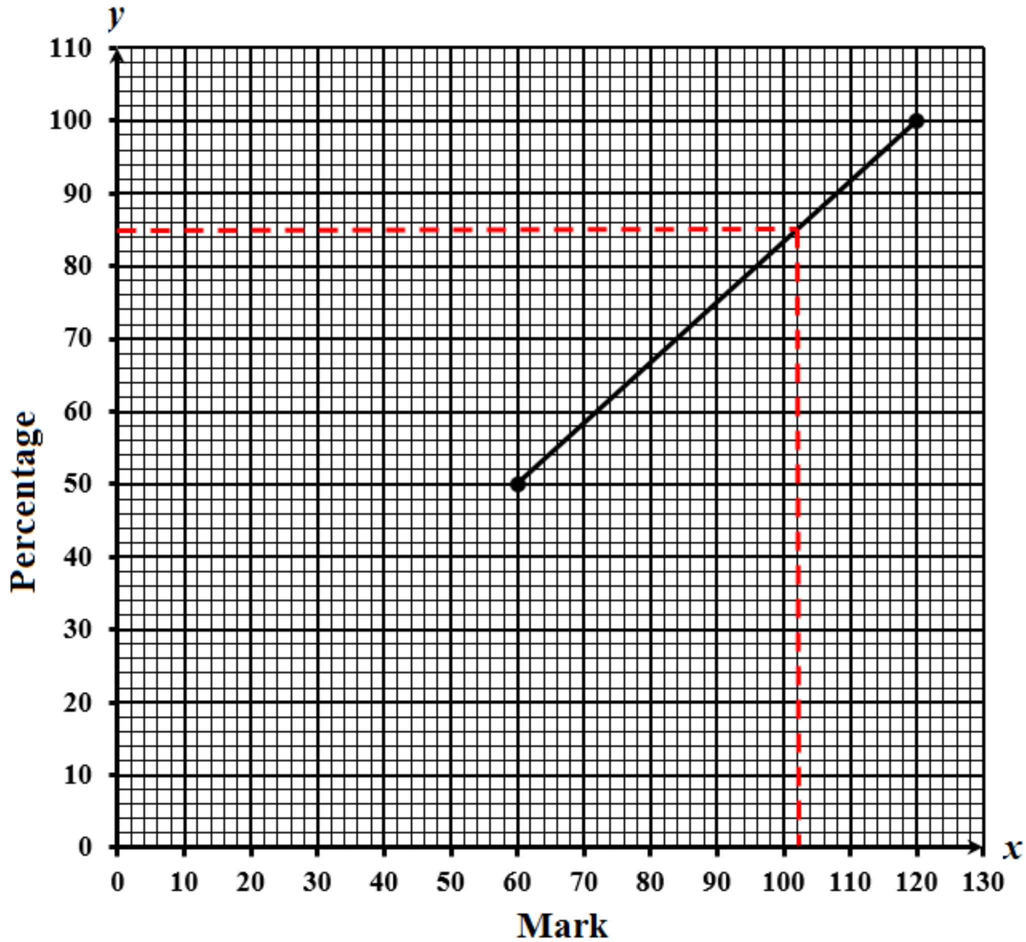
Percentage = 79% (obtained by a read off)

(iv) A candidate is awarded a Grade A if her percentage is 85% or more. Use the graph drawn in (ii) to determine the minimum mark the candidate needs to be awarded a Grade A.

Draw lines on your graph to show how the percentage was obtained.

SOLUTION:

Data: A candidate is awarded a Grade A if her percentage is 85% or more.
Required to find: The minimum mark the candidate needs to be awarded a Grade A
Solution:
Grade A is awarded for 85% or more.
Draw a horizontal at 85% to meet the graph.
At that point, we draw a vertical to meet the horizontal axis.
At this point a read off is made.



Minimum mark for a Grade A = 102, as shown in the above diagram

(b) The functions $f(x)$ and $g(x)$ are defined as

$$f(x) = 3x + 2 \qquad g(x) = \frac{x^2 - 1}{3}$$

(i) Evaluate $g(5)$.

SOLUTION:

Data: $f(x) = 3x + 2$ and $g(x) = \frac{x^2 - 1}{3}$

Required to calculate: $g(5)$

Calculation:

$$g(5) = \frac{(5)^2 - 1}{3}$$
$$= \frac{25 - 1}{3}$$
$$= \frac{24}{3}$$
$$= 8$$

(ii) Write an expression in terms of x for $f^{-1}(x)$.

SOLUTION:
Required to find: $f^{-1}(x)$
Solution:
$$f(x) = 3x + 2$$
Let $y = 3x + 2$
$$y - 2 = 3x$$
$$\frac{y-2}{3} = x$$
Replace y by x
$$f^{-1}(x) = \frac{x-2}{3}$$

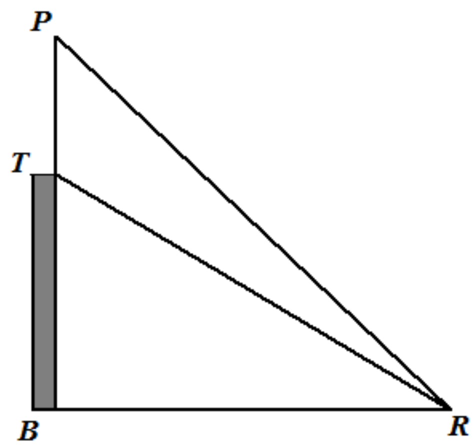
(iii) Write an expression for $gf(x)$, in the form $(x+a)(x+b)$, where a and $b \in R$.

SOLUTION:
Required to find: $gf(x)$ in the form $(x+a)(x+b)$, where a and $b \in R$
Solution:
$$gf(x) = \frac{(3x+2)^2 - 1}{3}$$
$$= \frac{9x^2 + 12x + 4 - 1}{3}$$
$$= \frac{9x^2 + 12x + 3}{3}$$
$$= 3x^2 + 4x + 1$$
$$= (3x+1)(x+1) \text{ or } 3\left(x + \frac{1}{3}\right)(x+1)$$

Note that $gf(x)$ cannot be expressed in the form $(x+a)(x+b)$ but, maybe it should have been, $c(x+a)(x+b)$, where $c=3$, $a=\frac{1}{3}$ and $b=1$

OR $(ax+b)(x+c)$ where $a = 3$, $b = 1$ and $c = 1$

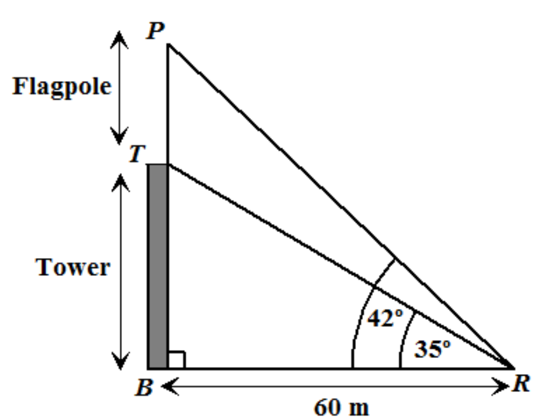
10. (a) The diagram below, **not drawn to scale**, shows a vertical tower, BT, with a flagpole, TP, mounted on it. A point R is on the same horizontal ground as B, such that $RB = 60$ m, and the angles of elevation of T and P from R are 35° and 42°, respectively.



(i) Label the diagram to show

- the distance 60 m
- the angles of 35 ° and 42°
- any right angle (s)

SOLUTION:
Data: The diagram below.
Required to label: The diagram given with the measurements
Solution:



(ii) Calculate the length of the flagpole, giving your answer to the nearest metre.

SOLUTION:

Required to calculate: The length of the flagpole.
Calculation:

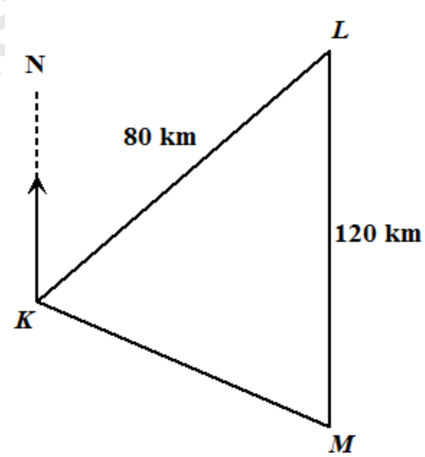
$$\tan 35° = \frac{BT}{60}$$
$$BT = 60 \tan 35°$$

$$\tan 42° = \frac{PB}{60}$$
$$PB = 60 \tan 42°$$

Length of the flagpole = Length of *PB* – Length of *BT*

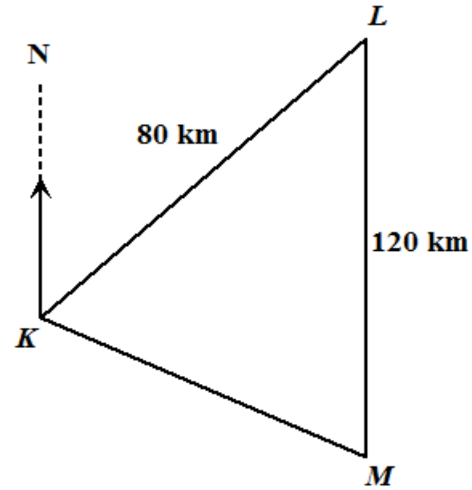
$$= 60 \tan 42° - 60 \tan 35°$$
$$= 12.0$$
$$= 12 \text{ m (to the nearest m)}$$

(b) The diagram below, **not drawn to scale**, shows the relative positions of three fishing boats, *K*, *L* and *M*. *L* is on a bearing of 040° from *K* and *M* is due South of *L*. $LM = 120$ km and $KL = 80$ km.

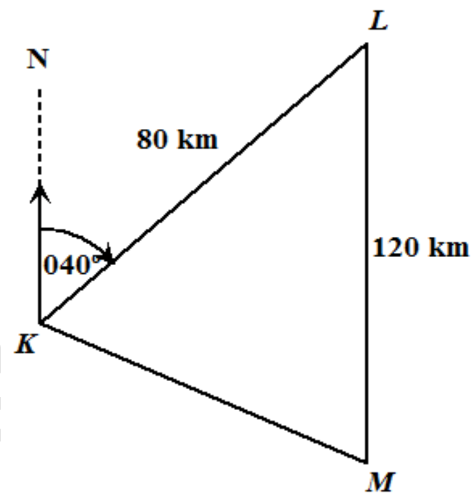


(i) On the diagram show the bearing of 040°.

SOLUTION:
Data: Diagram showing the position of the fishing boats. The bearing of *L* from *K* is 040° and *M* is south of *L*.



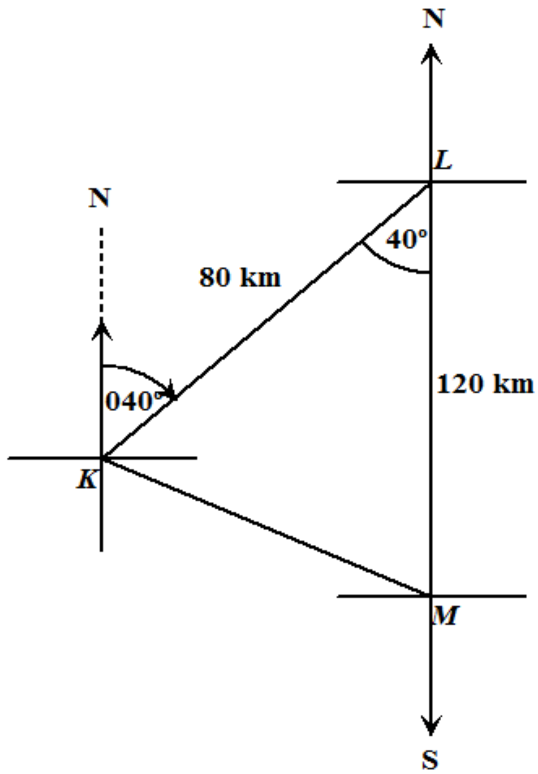
Required to show: The bearing of 040° on the diagram
Solution:



(ii) Calculate the measure of $\angle KLM$.

SOLUTION:

Required to calculate: $\angle KLM$
Calculation:



$K\hat{L}M = N\hat{K}L = 40°$ (Alternate angles)

(iii) Calculate the length, to the nearest kilometre, of *KM*.

SOLUTION:

Required to calculate: *KM*
Calculation:
Applying the cosine law to triangle *KLM*

$$\begin{aligned} KM^2 &= (80)^2 + (120)^2 - 2(80)(120)\cos 40° \\ &= 6400 + 14400 - 160(120)\cos 40° \\ &= 20800 - 160(120)\cos 40° \\ KM &= \sqrt{6091.95} \\ &= 78.\underline{\underline{0}} \\ &= 78 \text{ km (to the nearest km)} \end{aligned}$$

(iv) Calculate the measure of $\angle LKM$ to the nearest degree.

SOLUTION:

Required to calculate: $\angle LKM$
Calculation:
Applying the sine law to triangle *KLM*

$$\frac{120}{\sin L\hat{K}M} = \frac{KM}{\sin 40^\circ}$$

$$\therefore \sin L\hat{K}M = \frac{120 \times \sin 40^\circ}{78.0}$$

$$= 0.9889$$

$$L\hat{K}M = \sin^{-1}(0.9889)$$

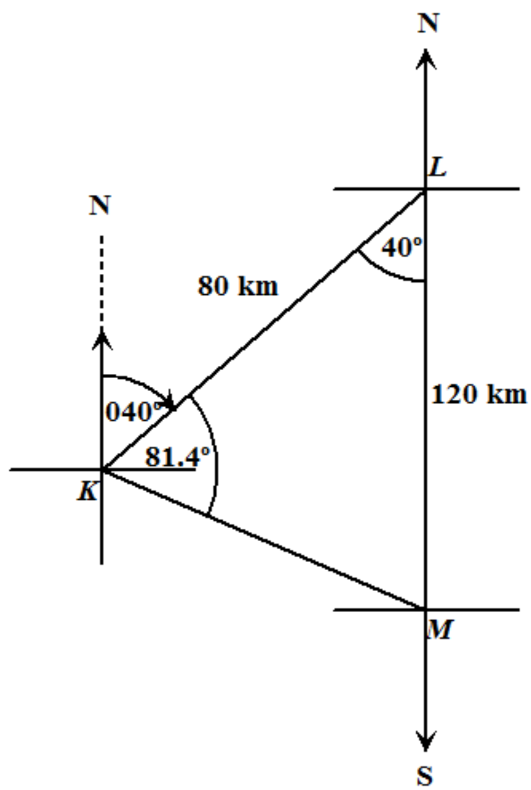
$$= 81.4^\circ$$

$$= 81^\circ \text{ (to the nearest degree)}$$

(v) Calculate the bearing of *M* from *K*.

SOLUTION:

Required to calculate: The bearing of *M* from *K*
Calculation:



The bearing of *M* from *K* $= 40^\circ + 81.4^\circ$ (the angle *NKM* on the diagram)
$= 121.4^\circ$

11. (a) (i) Calculate the matrix product **AB** where $\mathbf{A}=\begin{pmatrix}1 & 1\\ 2 & 3\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$.

SOLUTION:

Data: $\mathbf{A}=\begin{pmatrix}1 & 1\\ 2 & 3\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$

Required to calculate: AB

Calculation:

$A\times B$

$$\underset{2\times 2}{\begin{pmatrix}1 & 1\\ 2 & 3\end{pmatrix}}\underset{2\times 2}{\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}}=\underset{2\times 2}{\begin{pmatrix}e_{11} & e_{12}\\ e_{21} & e_{22}\end{pmatrix}}$$

$e_{11}=(1\times 1)+(1\times 0)=1$

$e_{12}=(1\times 2)+(1\times 1)=3$

$e_{21}=(2\times 1)+(3\times 0)=2$

$e_{22}=(2\times 2)+(3\times 1)=7$

$\therefore\ \mathbf{AB}=\begin{pmatrix}1 & 3\\ 2 & 7\end{pmatrix}$

(ii) Show that the matrix product of **A** and **B** is NOT commutative, that is, $\mathbf{AB}\neq\mathbf{BA}$.

SOLUTION:

Required to prove: $\mathbf{AB}\neq\mathbf{BA}$

Proof:

$\mathbf{B}\times\mathbf{A}$

$$\underset{2\times 2}{\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}}\underset{2\times 2}{\begin{pmatrix}1 & 1\\ 2 & 3\end{pmatrix}}=\underset{2\times 2}{\begin{pmatrix}e_{11} & e_{12}\\ e_{21} & e_{22}\end{pmatrix}}$$

$e_{11}=(1\times 1)+(2\times 2)=5$

Notice, e_{11} of **AB** = 1 and e_{11} of **BA** = 5.

There is no need to complete the entire multiplication for **BA,** since all the elements of **AB** will not be equal to all the corresponding elements of **BA**.

$\therefore\ \mathbf{AB}\neq\mathbf{BA}$

Q.E.D.

(iii) Find, $\mathbf{A}^{-1}$, the inverse the **A**.

SOLUTION:

Required to find: $\mathbf{A}^{-1}$
Solution:

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

$$\det A = (1\times 3) - (1\times 2)$$
$$= 1$$

$$\therefore A^{-1} = \frac{1}{1}\begin{pmatrix} 3 & -(1) \\ -(2) & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$$

(iv) Given that $\mathbf{M} = \begin{pmatrix} 2x & 2 \\ 9 & 3 \end{pmatrix}$, calculate the value(s) of x for which $|\mathbf{M}| = 0$.

SOLUTION:
Data: $\mathbf{M} = \begin{pmatrix} 2x & 2 \\ 9 & 3 \end{pmatrix}$ and $|\mathbf{M}| = 0$.
Required to calculate: x
Calculation:

$$|\mathbf{M}| = 0$$
$$\therefore (2x\times 3) - (2\times 9) = 0$$
$$6x - 18 = 0$$
$$6x = 18$$
$$\div 6$$
$$x = 3$$

(b) The position vectors of the points R, S and T, relative to an origin, O, are $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ respectively.

(i) Calculate the value of $\left|\overrightarrow{OR}\right|$.

SOLUTION:

Data: Position vectors of R, S and T, relative to the origin, O, are $\begin{pmatrix}-3\\4\end{pmatrix}$, $\begin{pmatrix}1\\1\end{pmatrix}$ and $\begin{pmatrix}5\\-2\end{pmatrix}$ respectively.

Required to calculate: $\left|\overrightarrow{OR}\right|$

Calculation:

$$OR = \begin{pmatrix}-3\\4\end{pmatrix}$$

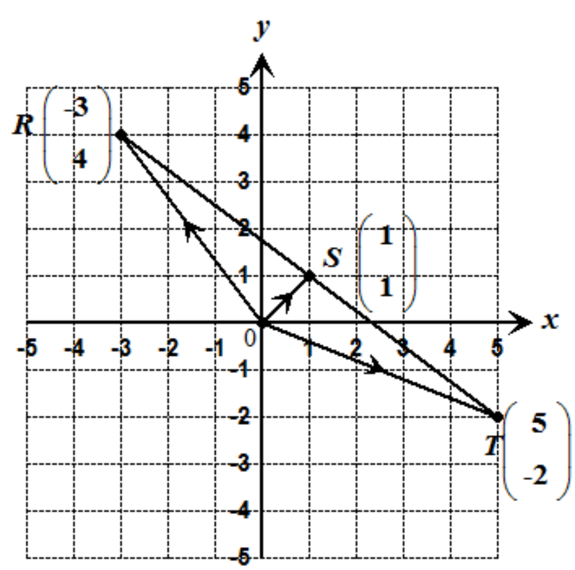
$$\begin{aligned}|OR| &= \sqrt{(-3)^2 + (4)^2}\\ &= \sqrt{25}\\ &= 5 \text{ units}\end{aligned}$$

(ii) Express in the form $\begin{pmatrix}x\\y\end{pmatrix}$, the vectors $\overrightarrow{RS}$ and $\overrightarrow{ST}$.

SOLUTION:

Required to express: $\overrightarrow{RS}$ and $\overrightarrow{ST}$ in the form $\begin{pmatrix}x\\y\end{pmatrix}$

Solution:



$$RS = RO + OS$$
$$= -\begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 4 \\ -3 \end{pmatrix} \text{ is of the form } \begin{pmatrix} x \\ y \end{pmatrix}, \text{ where } x = 4 \text{ and } y = -3.$$

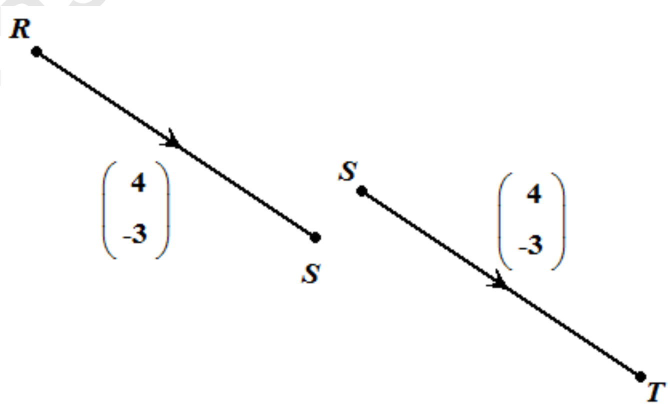
$$ST = SO + OT$$
$$= -\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$
$$= \begin{pmatrix} 4 \\ -3 \end{pmatrix} \text{ is of the form } \begin{pmatrix} x \\ y \end{pmatrix}, \text{ where } x = 4 \text{ and } y = -3.$$

(iii) Using the results of combining the vectors in (b) (ii). Justify that *RS* is parallel to *ST* and that *RST* is a straight line.

SOLUTION:

Required to prove: *RS* is parallel to *ST* and *RST* is a straight line. In other words, we are required to prove that *R*, *S* and *T* are collinear

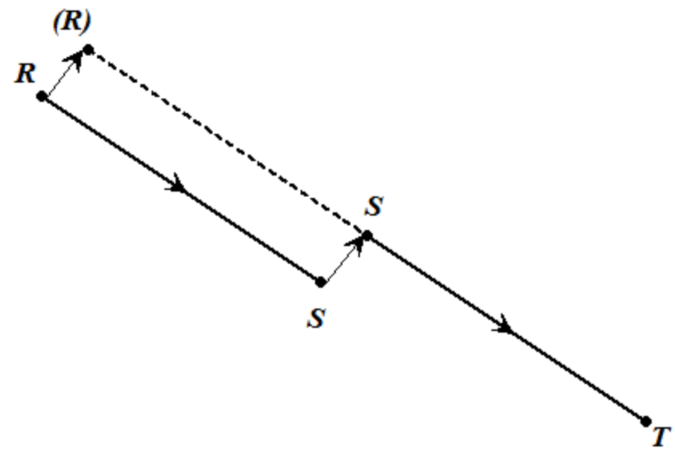
Proof:



$$RS = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$
$$ST = \begin{pmatrix} 4 \\ -3 \end{pmatrix} = 1 \times RS$$

$\therefore \overrightarrow{ST}$ is a scalar multiple, (1), of $\overrightarrow{RS}$ and so $\overrightarrow{ST}$ is parallel to $\overrightarrow{RS}$.

But S is a common point on both vectors.



Hence, R, S and T lie on the same straight line. (This is called collinearity)