FAS-PASS Maths

CXC MATHEMATICS JANUARY 2014
Section I

1. (a) Required to calculate: The exact value of: $\left(1\frac{3}{4}-\frac{1}{8}\right)+\left(\frac{5}{6}\div\frac{2}{3}\right)$.

Calculation:
Firstly, we calculate the part that is within the first set of brackets to obtain

$$\begin{aligned}1\frac{3}{4}-\frac{1}{8}&=\frac{7}{4}-\frac{1}{8}\\&=\frac{14}{8}-\frac{1}{8}\\&=\frac{13}{8}\end{aligned}$$

Secondly, we calculate the part that is within the second set of brackets to obtain

$$\begin{aligned}\frac{5}{6}\div\frac{2}{3}&=\frac{5}{\cancel{6}^{2}}\times\frac{\cancel{3}}{2}\\&=\frac{5}{4}\end{aligned}$$

Hence, the calculation of

$$\begin{aligned}\left(1\frac{3}{4}-\frac{1}{8}\right)+\left(\frac{5}{6}\div\frac{2}{3}\right)&=\frac{13}{8}+\frac{5}{4}\\&=\frac{13}{8}+\frac{10}{8}\\&=\frac{23}{8}\text{ or }2\frac{7}{8}\text{ (in exact form)}\end{aligned}$$

(b) Required to calculate: $\sqrt{2.891}+\frac{1.2}{(1.31)^2}$ **correct to 2 decimal places**

Calculation:
Ordinary arithmetic will be too cumbersome, so, we use the calculator

$$\begin{aligned}\sqrt{2.891}+\frac{1.2}{(1.31)^2}&=1.700+\frac{1.2}{1.716}\text{ (by the calculator)}\\&=1.700+0.699\\&=2.39\underline{\underline{9}}\\&=2.40\text{ (correct to 2 decimal places)}\end{aligned}$$

(c) **Data:** The price paid for 165 bracelets in China is \$6 800.
The amount paid in duty is \$1 360

(i) **Required to calculate:** The total cost of the bracelets, inclusive of duty
Calculation:

The price paid for the bracelets in China = \$6 800
+
The amount paid in duty for the bracelet = \$1 360
Therefore the total cost = \$8 160

$\therefore$ The total cost of the bracelets, inclusive of duty is \$8 160.

(ii) **Data:** The selling price of each bracelet is \$68.85.

a) **Required to calculate:** The total profit made on all the bracelets.
Calculation:

$$\text{The selling price of all 165 bracelets} = \$68.85 \times 165$$
$$= \$11360.25$$

The selling price exceeds the cost price so there is a profit

$$\begin{aligned}\text{Profit} &= \text{Selling price} - \text{Total cost} \\ &= \$11360.25 - \$8160.00 \\ &= \$3\,200.25\end{aligned}$$

b) **Required to calculate:** The profit as a percentage of the cost price.
Calculation:

Profit as a percentage of the total cost

$$= \frac{\text{Profit}}{\text{Total cost}} \times 100\,\%$$

$$= \frac{\$3\,200.25}{\$8160.00} \times 100$$

$$= 39.\underline{\underline{2}}\%$$

$$= 39\% \text{ (expressed to the nearest whole number)}$$

In (c) (ii) b), the percentage profit is always calculated as a percentage of the cost price of the item and so the term 'of the cost price' need not have been written in the question.

2. (a) (i) Data: $2(x-6)+3x \le 8,\ x \in \Re$

Required to solve for *x*

Solution:

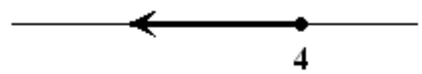
We solve the inequality much the same as solving an equation.
First, we expand and then simplify.

$$2(x-6)+3x \le 8$$
$$2x-12+3x \le 8$$

$$2x+3x-12 \le 8$$
$$2x+3x \le 8+12$$
$$5x \le 20$$
$$\div 5$$
$$x \le 4$$

(ii) Required to show: The solution to (a) (i) on a number line.

Solution:



We colour the point 4 to show that 4 is a part of the solution. The arrow is drawn to the left of 4 as x is less than or equal to 4.

(b) (i) Required to factorise: $3x-6y+ax-2ay$

Solution:

$3x-6y+ax-2ay$

Grouping the terms in two pairs so as to identify the common factors in the pairs we get:

$$\underbrace{3x-6y}+\underbrace{ax-2ay}=3\left(\underline{x-2y}\right)+a\left(\underline{x-2y}\right)$$

We factored out '3' from the first group and 'a' from the second group, as shown.

Now we can factor out the common term, $(x-2y)$ so as to obtain

$$3x-6y+ax-2ay=(x-2y)(3+a)$$

(ii) Required to factorise: p^2-1

Solution:

We re-write p^2-1 as shown below

$$p^2-1=(p)^2-(1)^2$$

This is now of the form of a 'difference of two squares'

Hence, $p^2-1=(p+1)(p-1)$

(c) **Required to expand and simplify** $(2k-3)(k-2)$

Solution:

$$(2k-3)(k-2) = 2k(k-2) - 3(k-2)$$
$$= 2k^2 - 4k - 3k + 6 \quad \text{(when completely expanded)}$$
$$= 2k^2 - 7k + 6$$
(when the terms in k are simplified)

(d) **Data:** The equation of two straight lines are $3x + y = 2$ and $4x - 2y = 6$.
The lines intersect at (x, y).

Required to prove: The point of intersection is $(1, -1)$

Proof:

Let

$3x + y = 2 \quad \ldots(1)$
$4x - 2y = 6 \quad \ldots(2)$

If $(1, -1)$ is the point of intersection of both lines, then $x = 1$ and $y = -1$ when substituted, should satisfy both equations.

Substituting $x = 1$ and $y = -1$ in equations (1) and (2), we obtain,

$$3(1) + (-1) = 2$$
$$3 + (-1) = 2$$
$$3 - 1 = 2$$
$$2 = 2 \text{ (True)}$$

$$4(1) - 2(-1) = 6$$
$$4 - 2(-2) = 6$$
$$4 + 2 = 6$$
$$6 = 6 \text{ (True)}$$

The substitution proves true for both equations.
Hence, $x = 1$ and $y = -1$ satisfy both equations and is therefore the point of intersection of the two lines, with the equations given.

Q.E.D.

An alternative solution, we can solve the equations of the two lines simultaneously, to obtain *x*= 1 and *y* = -1.

Using the method of elimination to solve the given equations

$3x + y = 2 \quad \ldots(1)$
$4x - 2y = 6 \quad \ldots(2)$

Equation (2) $\div 2$
$2x - y = 3 \quad \ldots(3)$

Equation (1) + Equation (3) will give

$$\begin{array}{r} 3x+y=2 \ + \\ \underline{2x-y=3} \\ \underline{5x=5} \end{array} \quad \text{(the term in } y \text{ is eliminated)}$$

$\div 5$

$$x=1$$

We can now substitute $x=1$ into either equation (1) or (2) to obtain the value of y.

Let us substitute $x=1$ in equation (1) to obtain

$$3(1)+y=2$$
$$3+y=2$$
$$y=2-3$$
$$y=-1$$

$\therefore x=1$ and $y=-1$

$\therefore (x,\ y)$, the point of intersection is $(1,\ -1)$.

Q.E.D.

We could also have solved the pair of equations simultaneously by the method of substitution, the graphical method or the matrix method. Let us see these other methods.

Using the substitution method:

$$3x+y=2 \qquad \ldots(1)$$
$$4x-2y=6 \qquad \ldots(2)$$

Using equation (1), we make y the subject to get

$$y=2-3x$$

We substitute $y=2-3x$ into equation (2) to obtain one equation in one unknown. This is

$$4x-2(2-3x)=6$$
$$4x-4+6x=6$$
$$4x+6x=4+6$$
$$10x=10$$
$$x=1$$

We now substitute $x=1$ into equation (1) to solve for y.
(We could have used equation (2) if we wished)

$$3(1) + y = 2$$
$$3 + y = 2$$
$$y = 2 - 3$$
$$y = -1$$

$\therefore$ The point of intersection $(x,\ y)$ is $(1,\ -1)$.

Using the matrix method:

$$3x + y = 2 \qquad \ldots(1)$$
$$4x - 2y = 6 \qquad \ldots(2)$$

Considering the coefficients of x and of y in the given equations we write the given equations in a matrix form. We obtain

$$\begin{pmatrix} 3 & 1 \\ 4 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} \quad \ldots\text{matrix equation}$$

Let $A = \begin{pmatrix} 3 & 1 \\ 4 & -2 \end{pmatrix}$ the 2x2 matrix in the matrix equation. Then we find the inverse of A, denoted by A^{-1}

$$\det A = (3 \times -2) - (1 \times 4)$$
$$= -6 - 4$$
$$= -10$$
$$\therefore A^{-1} = -\frac{1}{10}\begin{pmatrix} -2 & -(1) \\ -(4) & 3 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{2}{10} & \frac{1}{10} \\ \frac{4}{10} & -\frac{3}{10} \end{pmatrix}$$

Remember when the inverse of a matrix is multiplied by the matrix, we obtain the identity matrix and when the identity matrix is multiplied by any matrix, we get back the same matrix.

Now, we need to multiply both sides of the matrix equation by A^{-1} so as to obtain,

$$A^{-1} \times A \times \begin{pmatrix} x \\ y \end{pmatrix} = \underset{2\times\underline{\underline{2}}}{\begin{pmatrix} \frac{2}{10} & \frac{1}{10} \\ \frac{4}{10} & -\frac{3}{10} \end{pmatrix}} \underset{\underline{\underline{2}}\times 1}{\begin{pmatrix} 2 \\ 6 \end{pmatrix}}$$

$$2\times1$$

Therefore, the resulting matrix is of the form

$$=\begin{pmatrix} e_{11} \\ e_{21} \end{pmatrix}$$

$$\begin{aligned} e_{11} &= \left(\frac{2}{10}\times 2\right)+\left(\frac{1}{10}\times 6\right) \\ &= \frac{10}{10} \\ &= 1 \end{aligned}$$

$$\begin{aligned} e_{21} &= \left(\frac{4}{10}\times 2\right)+\left(-\frac{3}{10}\times 6\right) \\ &= -\frac{10}{10} \\ &= -1 \end{aligned}$$

Both the L.H.S and the R.H.S of the equation are 2x1 matrices which are equal.

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

So, equating corresponding entries, we obtain $x=1$ and $y=-1$.

$\therefore$ The point of intersection, (x, y) is $(1, -1)$.

Q.E.D.

Using the graphical method:

$$\begin{aligned} 3x+y&=2 \quad \ldots(1) \\ 4x-2y&=6 \quad \ldots(2) \end{aligned}$$

We treat both equations as straight line graphs and draw both on the same axes. The point of intersection will be the solution of the equations.

$3x+y=2$

x	**y**
0	2
-1	5

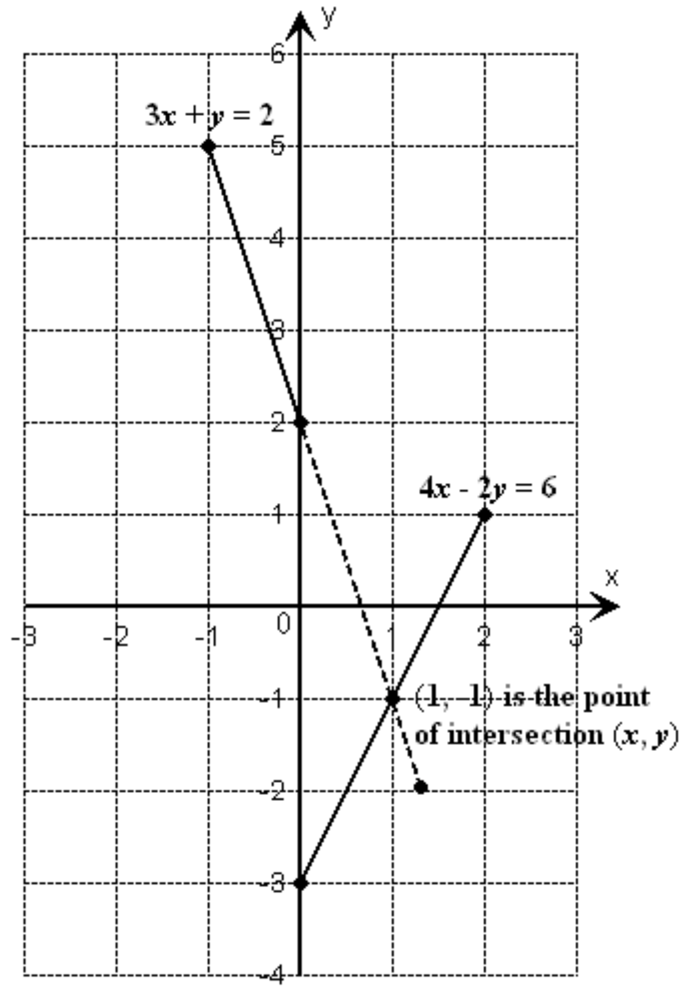
So, $(0, 2)$ and $(-1, 5)$ are two points on the straight line with the equation $3x+y=2$.

$4x-2y=6$

x	**y**
0	-3
2	1

So, $(0, -3)$ and $(2, 1)$ are two points on the straight line with the equation $4x-2y=6$.

We now draw both lines on the same axes and extend either or both lines, only if necessary, to obtain the point of intersection and which will be the solution.



The point of intersection is $(1, -1)$ and so $(x,\ y) = (1, -1)$.

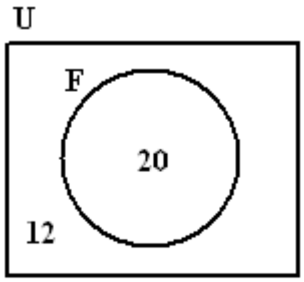
Q.E.D.

3. **Data:** Class has 32 students.
All students study Spanish (S).
20 students study French (F).

(i) **Required to show:** The data on a Venn diagram.
Solution:

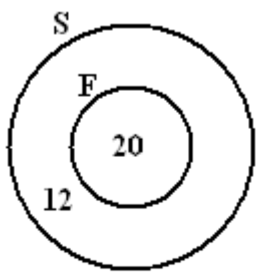
Let U = {Students in the class}
All the students study Spanish (S).
$\therefore$ U = S.
F is a subset of both U and S.

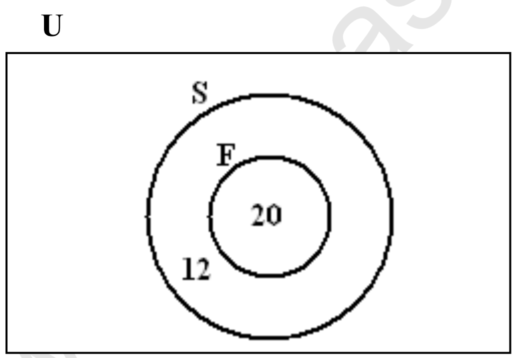
The number of students who study Spanish but not French will be $32 - 20 = 12$



OR

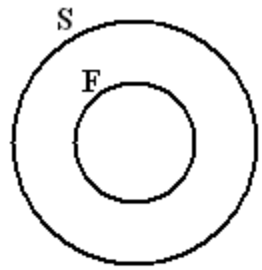
We could simply draw F as a subset of S and redefine the Universal set as say, the students in the school.





Either diagram would be suitable when properly defined

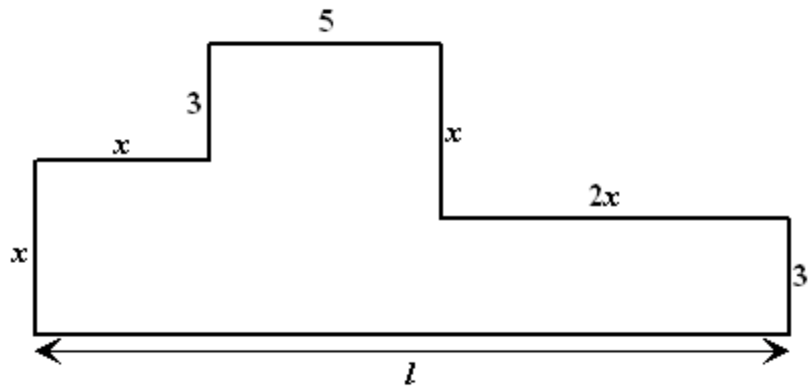
(ii) **Required to calculate:** The number of students who study Spanish but not French.
Calculation:
All 32 students of the class study Spanish.
Only 20 of these students study French.
Hence, 32 – 20 = 12 students study Spanish but do not study French.
$\therefore$ The number of students who study Spanish but not French = 12.



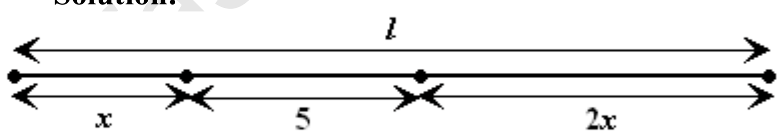
All the members of F belong to S but not all the members of S belong to F.
F is a subset of S and since F and S are not equal, we may even say that is a proper subset of S. This is illustrated in the above Venn diagram.

This can be expressed as, $F \subset S$

(b) **Data:** Diagram showing the plan of a floor.

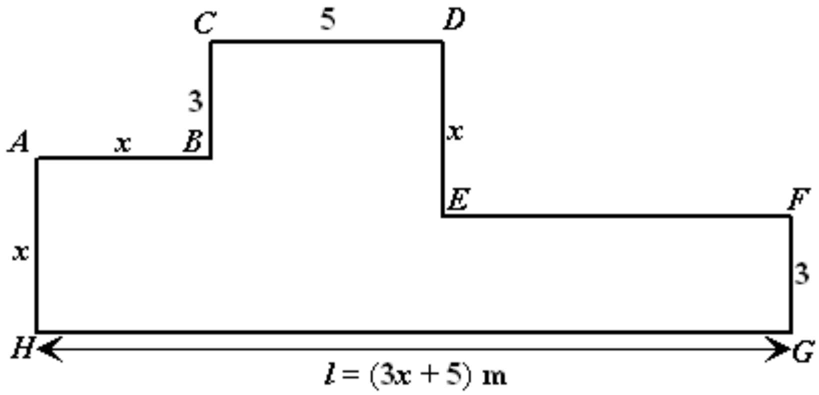


(i) **Required to state:** The length, *l*, of the floor.
Solution:



$l = x + 5 + 2x$ simplifying to give
$l = (3x + 5)$ m

(ii) **Data:** The perimeter of the floor = 56 m.
a) **Required to calculate:** The value of x.
Calculation:



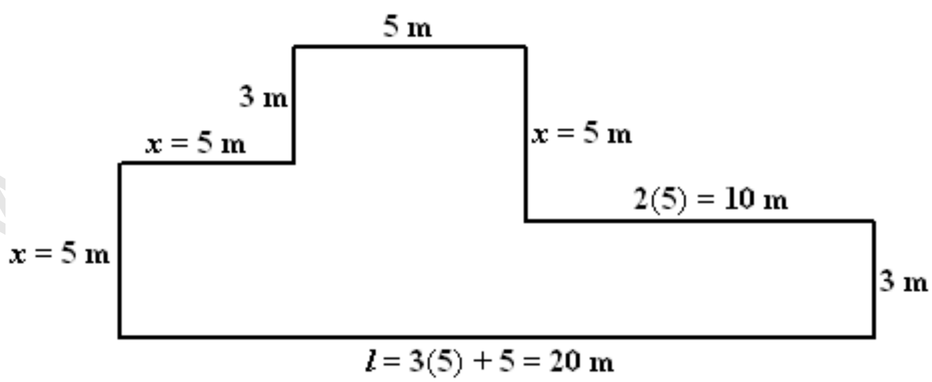
The perimeter of the figure is the sum of the lengths of all the sides of the figure. Let us start at A and obtain the perimeter by adding the lengths of all the sides to give

$$\begin{aligned}\text{Perimeter} &= x+3+5+x+2x+3+(3x+5)+x \\ &= x+x+2x+x+3x+5+3+5+3 \\ &= (8x+16)\text{ m}\end{aligned}$$

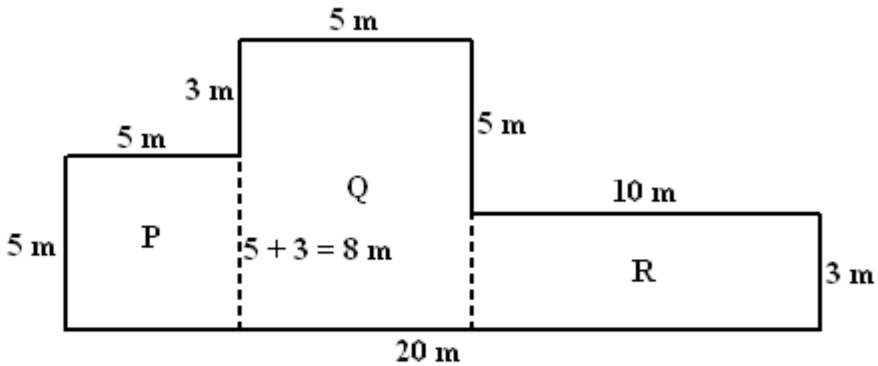
Hence,

$$\begin{aligned}8x+16 &= 56 \quad \text{(Data)} \\ \therefore 8x &= 56-16 \\ 8x &= 40 \\ \div 8 & \\ x &= 5\end{aligned}$$

b) **Required to calculate:** The area of the floor.
Calculation:



Since the shape is a compound, that is, it is composed of two or more simple shapes. So it is best that we first divide it into these simple shapes, as shown.



The floor is now divided into regions marked P, Q and R, as shown above.
The area of square, P $= 5 \times 5 = 25 \text{ m}^2$

The area of rectangle, Q $= 8 \times 5 = 40 \text{ m}^2$

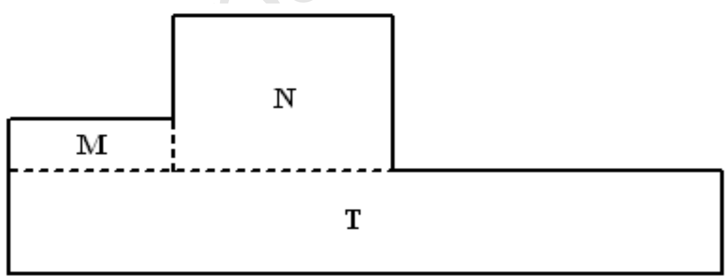
The area of rectangle, R $= 10 \times 3 = 30 \text{ m}^2$

The area of the entire floor = Area of P + Area of Q + Area of R

$$= (25 + 40 + 30) \text{ m}^2$$

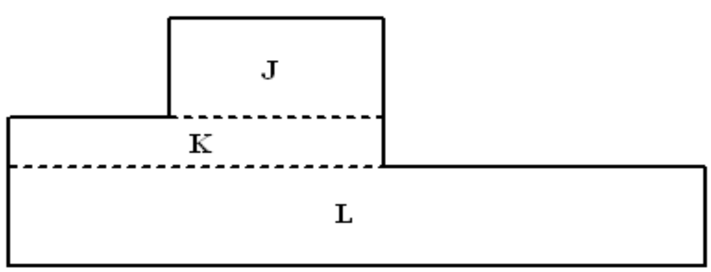
$$= 95 \text{ m}^2$$

It is useful to know that the floor could have been divided differently to that shown above and to still obtain the same area. For example, the floor is divided into the simple shapes M,N and T as illustrated.



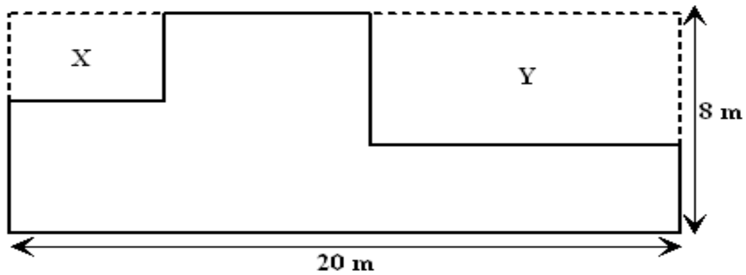
The area of floor = Area of M + Area of N + Area of T

OR



Area of floor = Area of J + Area of K + Area of L

OR

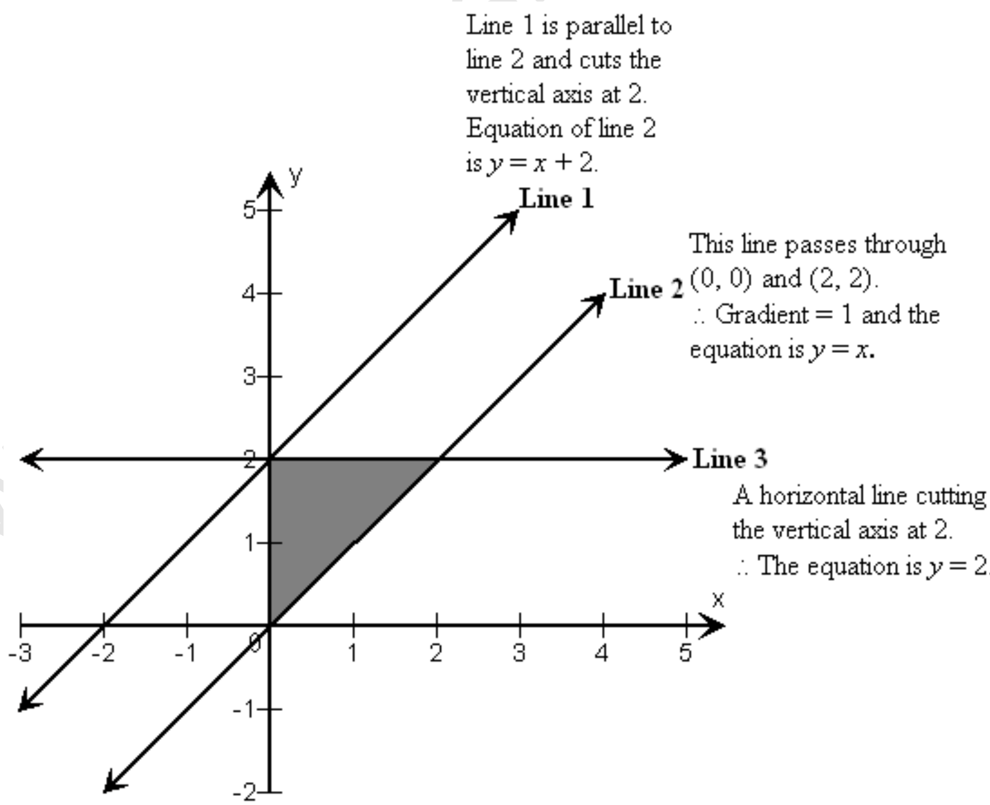


We could complete a 20 m x 8 m outer rectangle by adding regions X and Y as shown.
Area of the outer rectangle (20 m by 8 m) – (Area of X + Area of Y)

The data which indicated that the measurements were taken to the nearest metre was of no consequence in the working of any part of the question and could be 'misleading'. This is often the case with inconsequential mathematical data.

4. **Data:** A diagram showing a shaded region bounded by three straight lines, $y=2$, $y=x$ and $y=x+2$.

   (a) **Required to state:** The equation of line 1, line 2 and line 3.
   Solution:



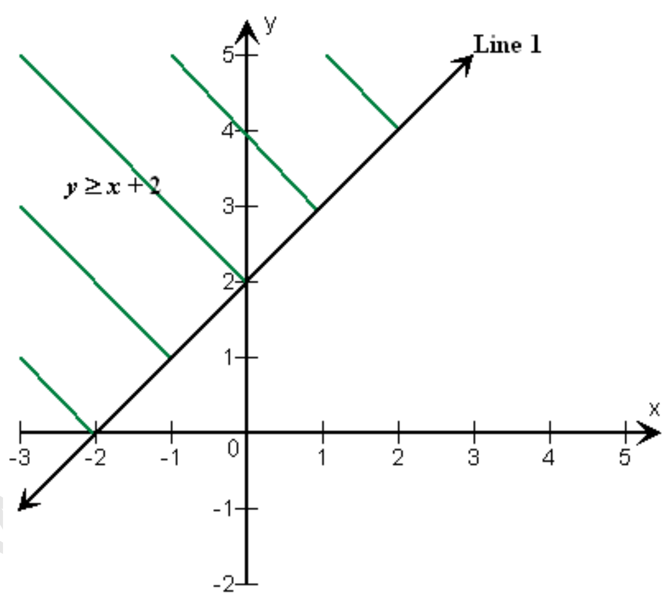
The equation of line 1 is $y = x + 2$ (gradient of 1 and cuts the vertical axis at 2)
The equation of line 2 is, $y = x$ (gradient of 1 and cuts the vertical axis at 0)
The equation of line 3 is, $y = 2$ (a horizontal line cutting the vertical axis at 2)

(b) **Required to prove:** The gradient of the line 2 is 1.
Proof:
Taking two points on line 2. We take $(0, 0)$ and $(2, 2)$.

$\therefore$ The gradient of line 2 is $\frac{2-0}{2-0} = \frac{2}{2}$
$= 1$

Q.E.D.

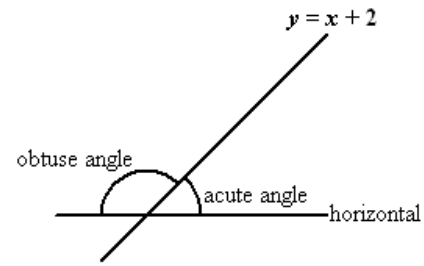
(c) **Required to shade:** The region described by, $y \geq x + 2$.
Solution:



We could choose a point and test the inequality
For example, let us choose the point (0,0) and test for validity in the inequality, $y \geq x + 2$.
We would obtain $0 \geq 0 + 2$ which is false.
Hence the region which lies opposite to that which contains the point, (0,0) is the region, $y \geq x + 2$.

OR

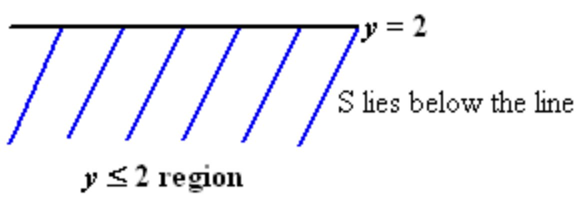
Alternatively, we may deduce the correct region by simply drawing a horizontal line to cut the line, $y = x + 2$

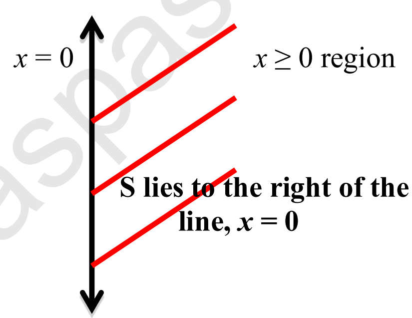


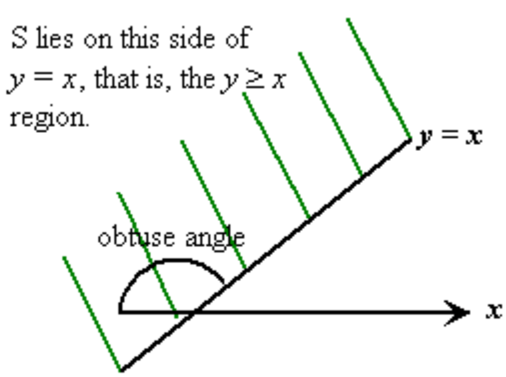
The side of the line which contains the obtuse angle identifies the region, $y \geq x + 2$.

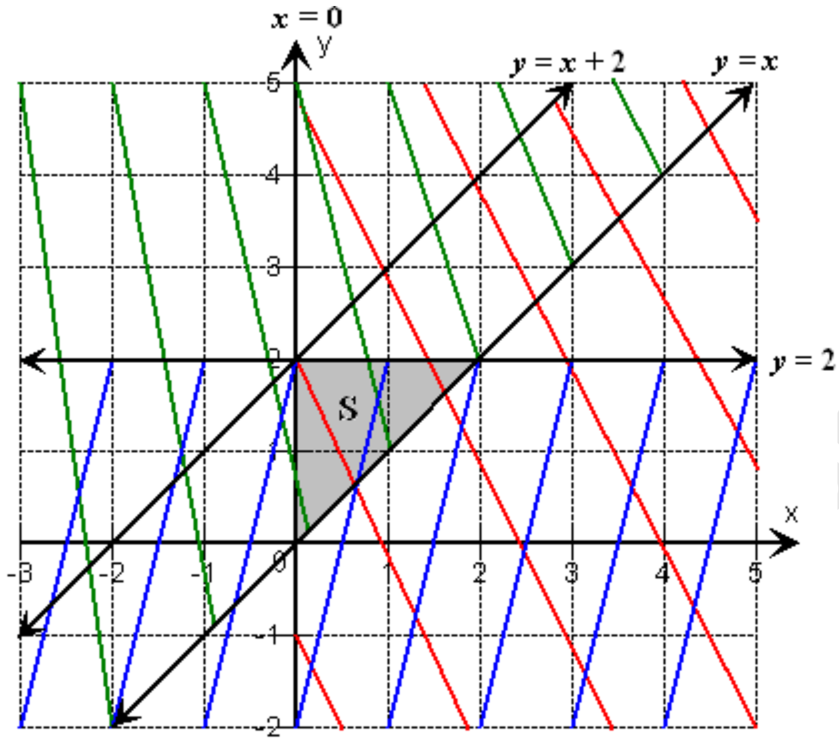
(d) **Required to state:** The three inequalities that define the shaded region, S.
Solution:

We look at each line as a boundary and observe the shaded region lying with respect to the line. This will help us to identify the inequality.



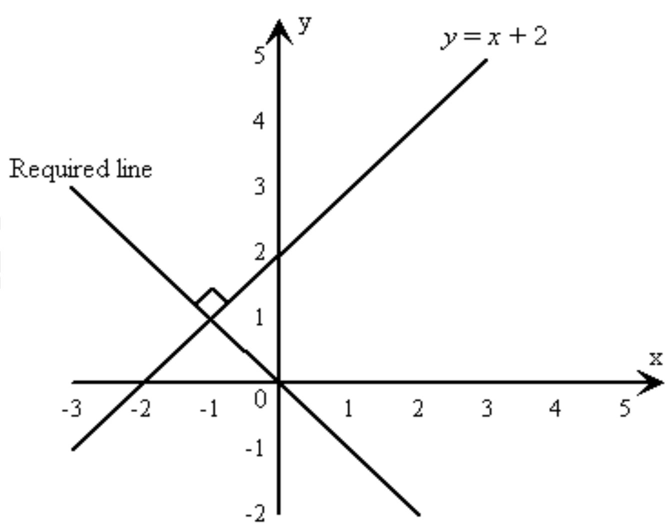






Putting these all together we obtain:
Region S is defined by $y \le 2$, $y \ge x$ and $x \le 0$.

(e) **Required to write:** The equation of the line that is perpendicular to line 1 and which passes through the origin.



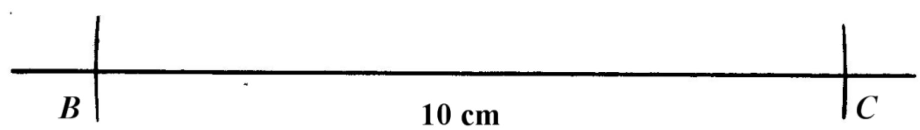
Solution:
Gradient of line 1, $y = x + 2$ is 1

(The equation of line 1 is of the form $y = mx + c$ where the gradient $m = 1$)
Therefore, the gradient of the required line, which is perpendicular to line 1 is
$\frac{-1}{1} = -1$,
(since the product of the gradients of perpendicular lines is -1)

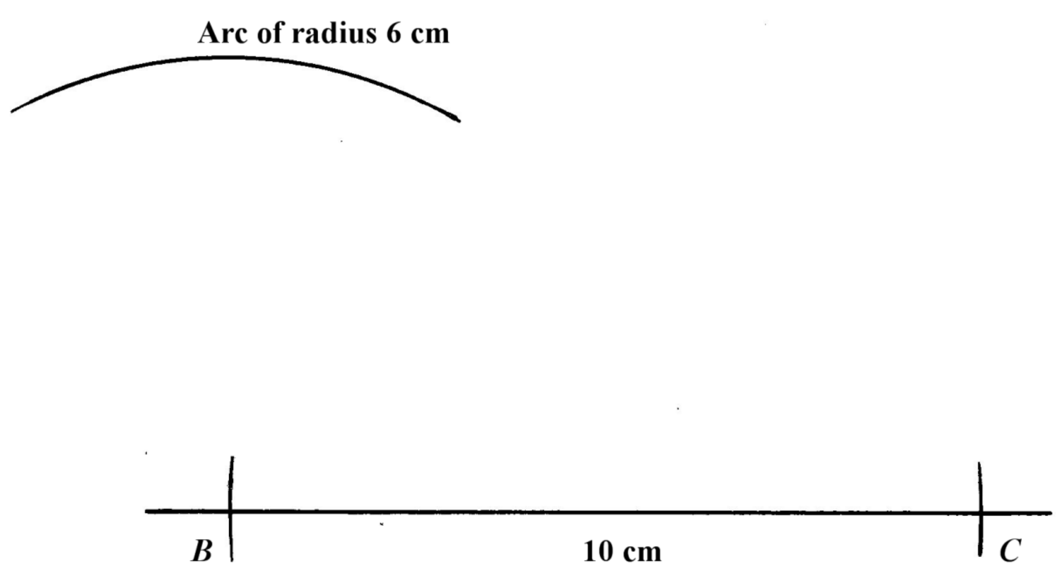
The required line passes through O.
The general equation of a straight line is $y = mx + c$ where m is the gradient and where c is the intercept on the y-axis.
Therefore, the equation of the required line can simply be stated as $y = -1(x) + 0$, that is, $y = -x$.

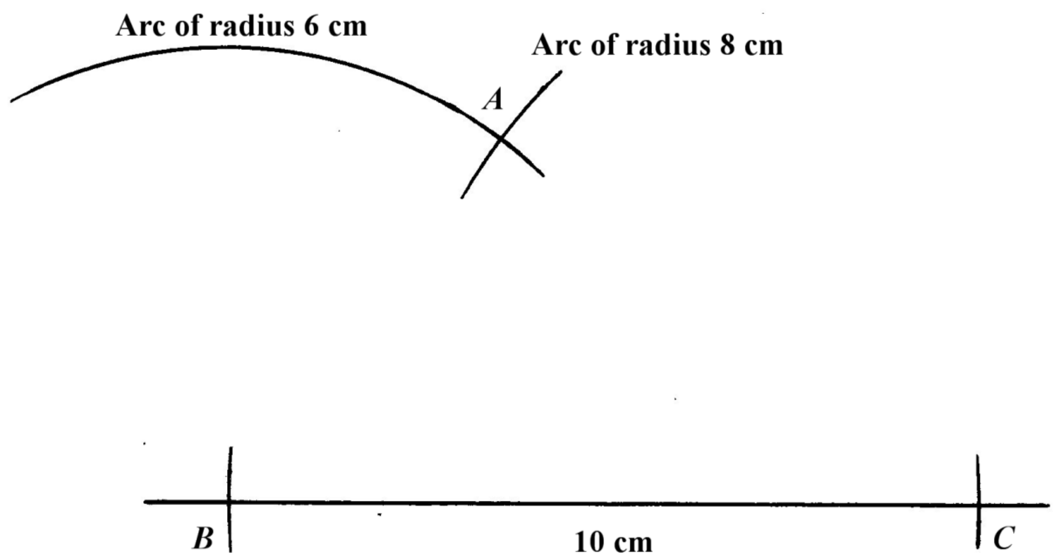
5. (a) (i) **Required to construct:** Triangle ABC, with $BC = 10$ cm, $AB = 6$ cm and $AC = 8$ cm.

Construction:
We draw a straight line longer than 10 cm and cut off BC 10 cm. The arc lines must be clearly shown

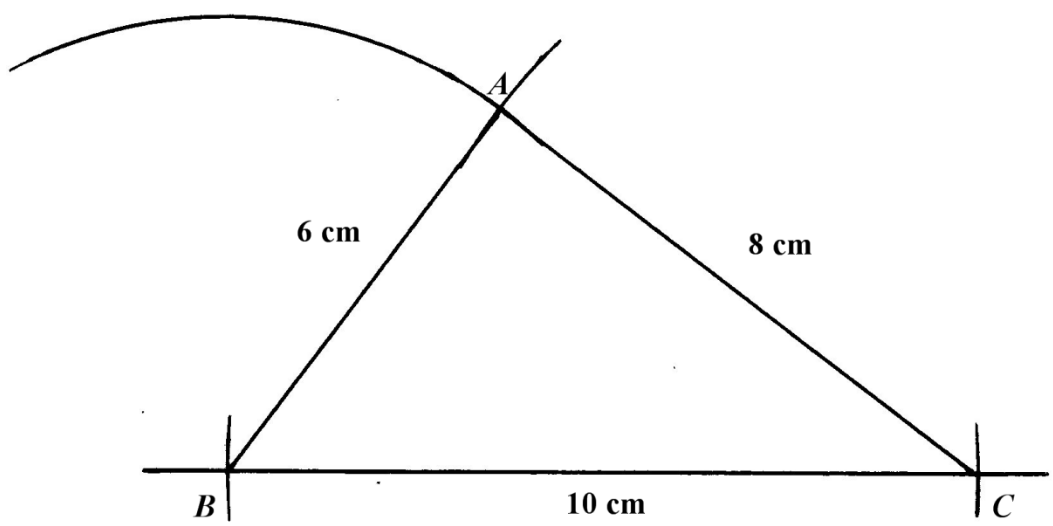


Now we draw arcs of radius 6 cm from B and radius 8 cm from C to intersect at A



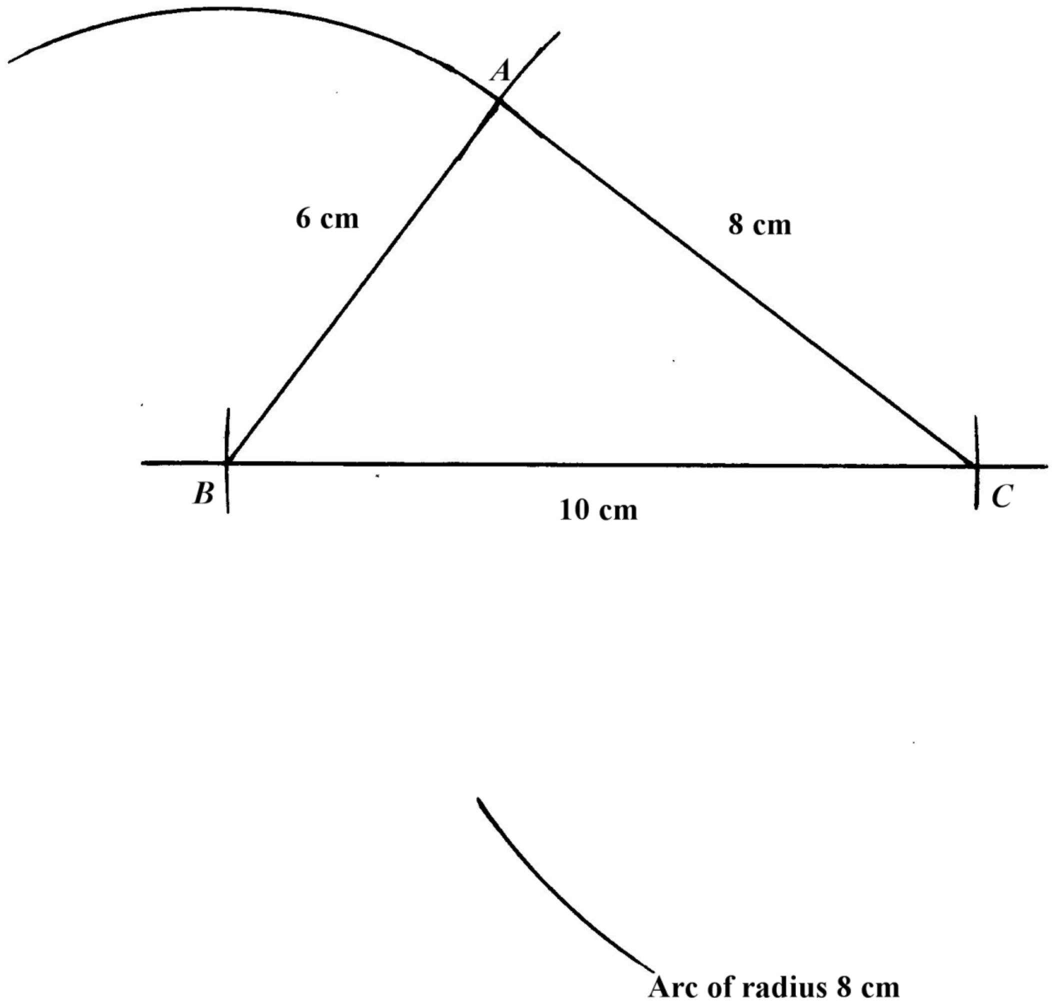


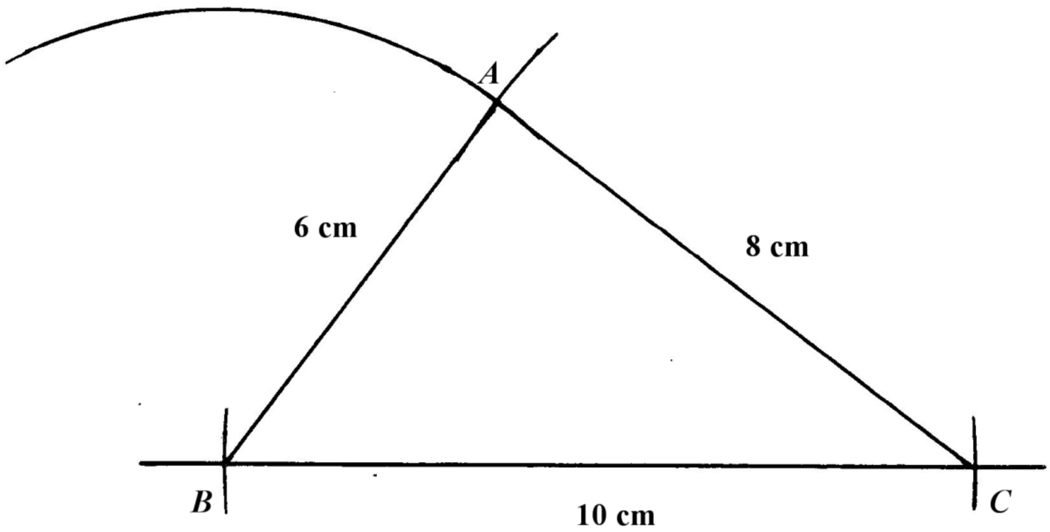
We join A to B and A to C to obtain the triangle ABC.

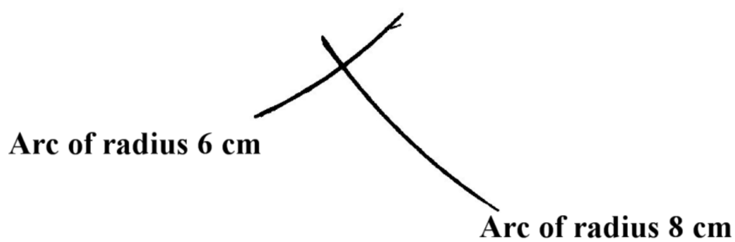


(ii) **Required to measure:** The size of angle ABC
Solution:
The size of angle $ABC = 53°$ (obtained by measurement using the protractor)

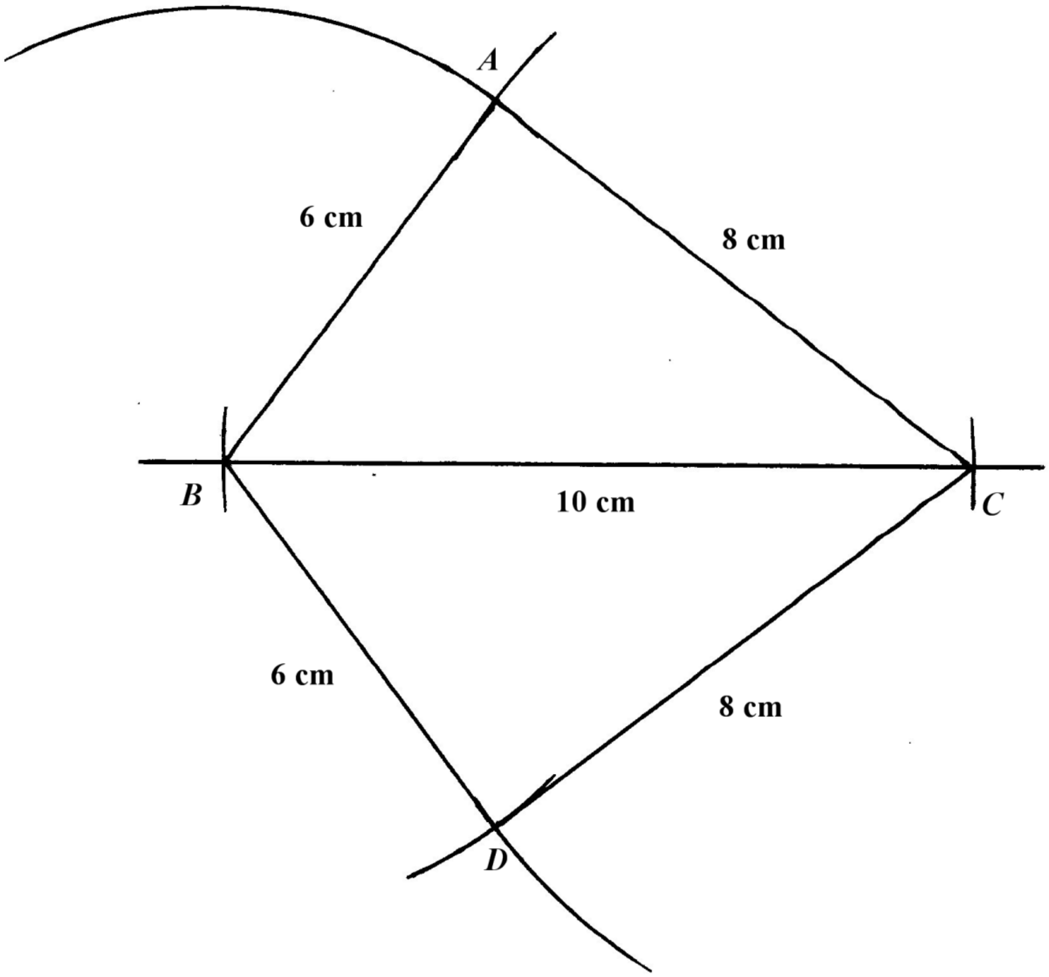
(iii) **Required To Complete:** The diagram to show a quadrilateral $CADB$, such that $CD = CA$ and $BD = BA$.
Construction:


A
6 cm
8 cm
B
10 cm
C
Arc of radius 8 cm



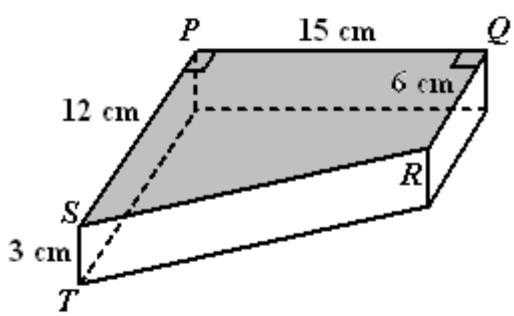


From *B* an arc of radius 6 cm is drawn and from *C* an arc of radius 8 cm is drawn so as to intersect at *D*. *BC* and *BD* are joined to complete the quadrilateral.

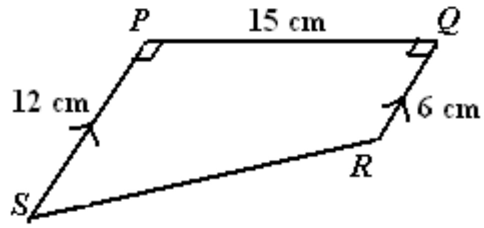


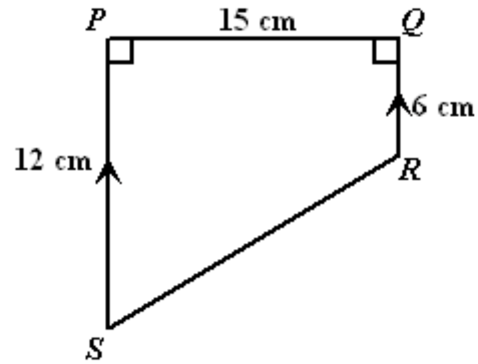
The quadrilateral *CABD* is a kite since two pairs of adjacent sides of the quadrilateral are equal in length

(b) **Data:** The diagram of a metal block with a trapezium cross section.



(i) **Required to calculate:** The area of the trapezium *PQRS*.
Calculation:





The area of the trapezium face is

$$=\frac{1}{2}\left(\text{Sum of the parallel sides}\times\text{Perpendicular distance between them}\right)$$

Therefore the area of trapezium, $PQRS=\frac{1}{2}\left(12+6\right)15$

$$=135\ \text{cm}^2$$

(ii) **Data:** The block is 3 cm thick.
Required to calculate: The volume of the block of metal
Calculation:
The volume of the block = Cross-Sectional area × perpendicular height or in this case the thickness.

$=\text{Cross-sectional area}\times 3$

$= 135 \text{ x } 3$

$= 405 \text{ cm}^2$

(iii) **Data:** The mass of the block is 1.5 kg.
Required to calculate: The mass of 1 cm^3 of the metal
Calculation:
The volume of the block = 405 cm^3
The mass of the block $=1.5\ \text{kg}$
Since 1 kg = 1 000 g, the mass of the block in g is

$=1.5\times 1000$

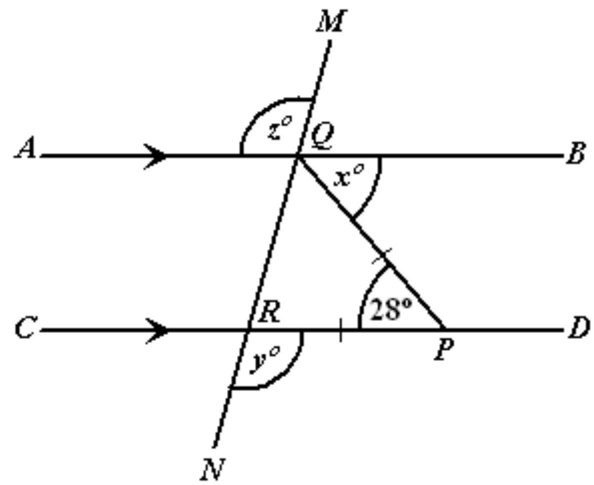
$=1500\ \text{g}$

The mass of 1 cm^3 of the metal $=\dfrac{1500}{405}\ \text{g}$

$=3.7\underline{\underline{0}}3$

$=3.7\ \text{g}\,(\text{to 1 decimal place})$

6. (a) **Data:** Diagram showing parallel lines cut by a transversal.

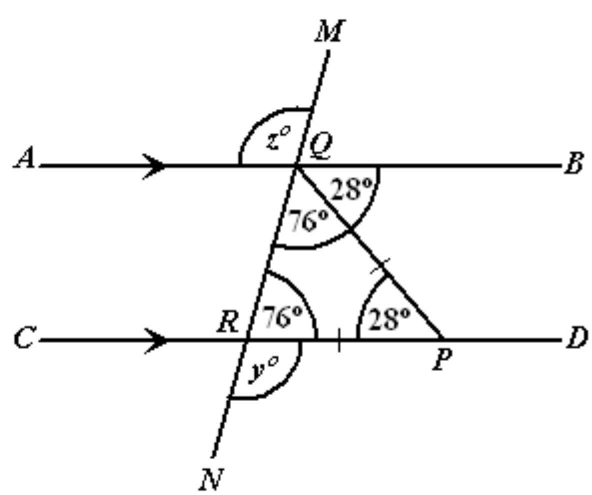


(i) **Required to calculate:** The value of x.
Calculation:
$x° = 28°$ (alternate angles are equal with $B\hat{Q}P$ and $Q\hat{P}R$ being alternate)
Hence, $x = 28$

(ii) **Required to calculate:** The value of y.
Calculation:
$PQ = PR$ (data)
$\therefore$ Triangle PQR is isosceles from the data
$P\hat{Q}R = P\hat{R}Q$ (since the base angles of an isosceles triangle are equal)

$$P\hat{Q}R = P\hat{R}Q = \frac{180° - 28°}{2}$$

(The sum of the interior angles of a triangle = 180°).
Hence, each of the angles will be $= \frac{152°}{2} = 76^0$



$y° + 76° = 180°$
(the sum of the angles at a point on a straight line totals 180^0)
$\therefore y° = 180° - 76°$
$= 104°$
Hence, $y = 104$

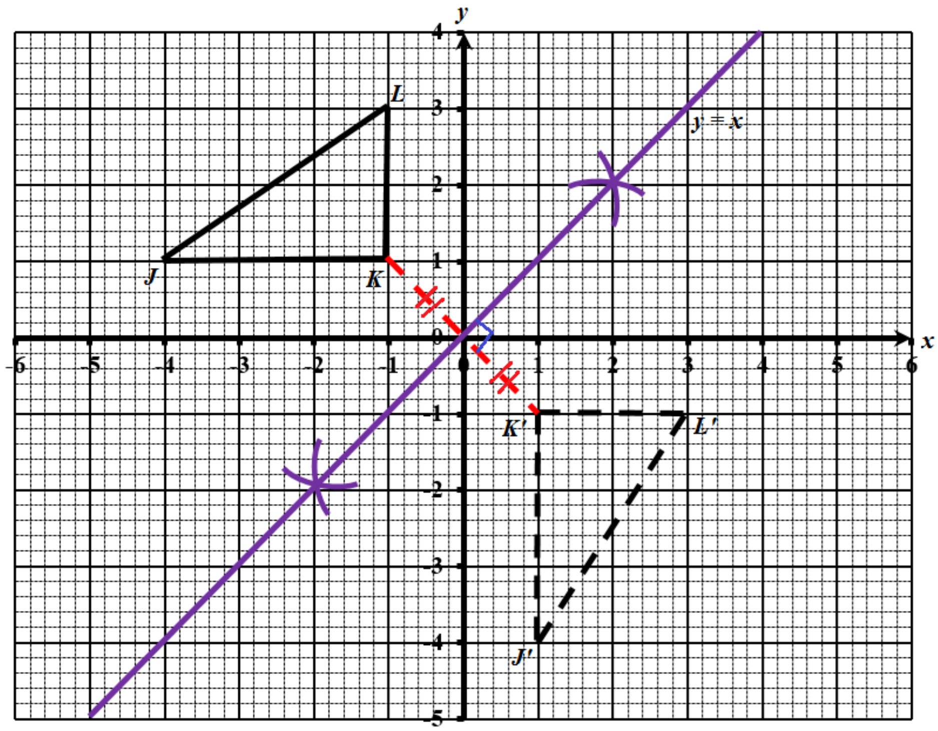
(iii) **Required to calculate:** The value of z
Calculation:
$Q\hat{R}C = 104°$
(Two straight lines intersect. So, vertically opposite angles to $P\hat{R}N$ are equal)

$A\hat{Q}M = 104°$
($Q\hat{R}C$ and $A\hat{Q}M$ are equal since they are corresponding angles)
$z° = 104°$
Hence, $z = 104$

(b) **Data:** Diagram showing two congruent triangles JKL and $J'K'L'$.
(i) **Required to state:** The coordinates of J.
Solution:
The point J has coordinates of $(-4, 1)$ (found by a read off from the given diagram).

(ii) **Required to state:** The length of $K'L'$.
Solution:
1 block = 1 cm (by measurement)
$\therefore K'L' = 2\,\text{cm}$ (since it is 2 blocks in length)

(iii) **Required to state:** The single transformation that maps JKL onto $J'K'L'$.
Solution: We observe that:
(i) Triangle JKL and triangle $J'K'L'$ are congruent.
(ii) The image $J'K'L'$ is laterally inverted with respect to its object triangle JKL.
We conclude that
the transformation is therefore a reflection.
$JKL \xrightarrow{\text{Reflection}} J'K'L'$



If we were to join K to K' (or J to J' or L to L') and construct the perpendicular bisector of any of these lines, this perpendicular line would be the same for any of these three constructions. This line would be the line of reflection.

As shown in the diagram, the perpendicular bisector is the line with equation $y = x$.

Therefore, the transformation can be described as a reflection in the line with equation $y = x$.

(iv) **Data:** Triangle JKL is mapped onto triangle $J''K''L''$ under the translation vector $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$.

Required to state: The coordinates of J'', K'' and L''.

Solution:

$$JKL \xrightarrow{\begin{pmatrix} 5 \\ -3 \end{pmatrix}} J''K''L''$$

The coordinates of $J = (4, -1)$

$$\therefore \begin{pmatrix} 4 \\ -1 \end{pmatrix} \xrightarrow{\begin{pmatrix} 5 \\ -3 \end{pmatrix}} \begin{pmatrix} -4+5 \\ 1+(-3) \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$\therefore J'' = (1, -2)$

The coordinates of $K = (-1,\ 3)$

$$\therefore \begin{pmatrix} -1 \\ 1 \end{pmatrix} \xrightarrow{\begin{pmatrix} 5 \\ -3 \end{pmatrix}} \begin{pmatrix} -1+5 \\ 1+(-3) \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$

$\therefore K'' = (4,\ -2)$

The coordinates of $L = (-1,\ 3)$

$$\therefore \begin{pmatrix} -1 \\ 3 \end{pmatrix} \xrightarrow{\begin{pmatrix} 5 \\ -3 \end{pmatrix}} \begin{pmatrix} -1+5 \\ 3+(-3) \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$\therefore L'' = (4,\ 0)$

We have found the coordinates of the three images under the given translation.

7. Data: A table showing the heights of a sample of seedlings.

To obtain the solutions for (a), (b) and (c) it is considered convenient to modify the table with the extra columns, as shown below.

LCL-Lower class limit UCL-Upper class limit
LCB – Lower class boundary UCB – Upper class boundary

Height, *x* (cm) **LCL-UCL**	**Lower Class Boundary LCB**	**Upper Class Boundary UCB**	**Midpoint or Mid-class Interval MCI**	**Frequency, *f***	**Points to be plotted** **(MCI , Frequency)**
					$(0,\ 0)$
3 – 7	$2.5 \le x < 7.5$		$\frac{2.5+7.5}{2} = 5$	0	$(5,\ 0)$
8 – 12	$7.5 \le x < 12.5$		$\frac{7.5+12.5}{2} = 10$	3	$(10,\ 3)$
13 – 17	$12.5 \le x < 17.5$		$\frac{12.5+17.5}{2} = 15$	12	$(15,\ 12)$
18 – 22	$17.5 \le x < 22.5$		$\frac{17.5+22.5}{2} = 20$	16	$(20,\ 16)$
23 – 27	$22.5 \le x < 27.5$		$\frac{22.5+27.5}{2} = 25$	22	$(25,\ 22)$
28 – 32	$27.5 \le x < 32.5$		$\frac{27.5+32.5}{2} = 30$	18	$(30,\ 18)$
33 – 37	$32.5 \le x < 37.5$		$\frac{32.5+37.5}{2} = 35$	14	$(35,\ 14)$
				$\sum f = 85$	$(40,\ 0)$

(a) **Required to find:** The number of seedlings in the sample.
Solution:
The number of seedlings in the sample is obtained from $\sum f = 85$, as seen in the table.

(b) **(i)** **Required to state:** The lower class limit for the class interval 8 – 12.
Solution:
For the class interval 8 – 12, 8 is the lower class limit and 12 is the upper class limit. Therefore, the lower class limit is 8 as seen in the above table and in the first column.

(ii) **Required to state:** The upper class boundary for the class interval 8 – 12.
Solution:

For the class interval 8 – 12, 7.5 is the lower class boundary and 12.5 is the upper class boundary.
Therefore, the upper class boundary is 12.5 as seen in the above table and on the right hand side of the second column and in the second row.

(iii) **Required To State:** The class width for the class interval 8 – 12.
Solution:
Class width or class interval = Upper class boundary – Lower class boundary

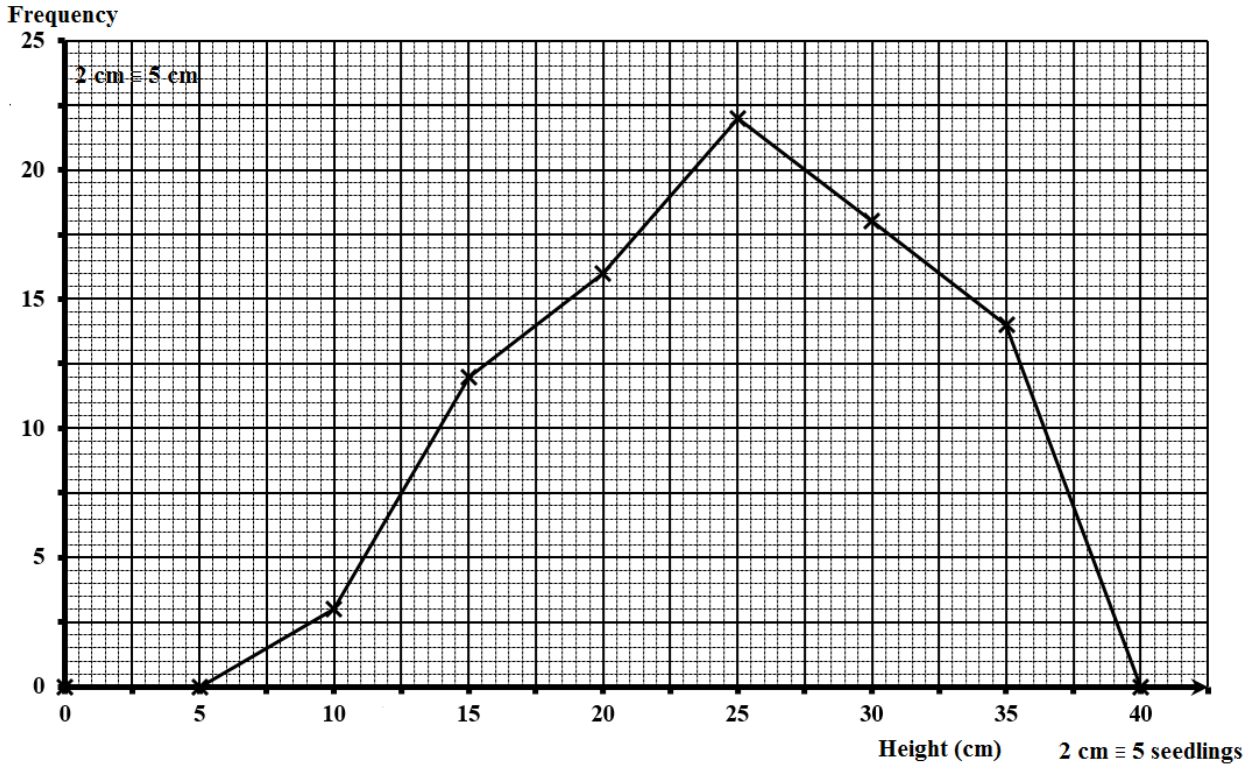
$$= 12.5 - 7.5$$
$$= 5$$

(c) (i) and (ii) have already been completed and shown in the modified table above.

The points to be plotted are shown in the last column of the modified table.
By checking backwards, we obtain and include the point $(0, 0)$ and by checking forward, we obtain and include the point, $(40, 0)$. These points are included on the frequency polygon so that the polygon is complete as being enclosed. It starts and ends on the horizontal axis.

(d) Required to use a scale of 2 cm to represent 5 cm on the horizontal axis and 2 cm to represent 5 seedlings on the vertical axis to draw a frequency polygon to represent the data shown on the table.

Solution:



Observations:

It should be noted that it is 'unusual' and frankly incorrect to have a first row in a table of data with a frequency of zero. The first row of the tabulated data should have been:

8 – 12	$7.5 \leq x < 12.5$	$\frac{7.5+12.5}{2}=10$	3	$(10, 3)$

The point, (5,0) should now be the initial or starting point of the frequency polygon and obtained when we check backwards to locate the starting point.

In (c) (ii) There cannot be expected to have 'missing values' after the interval '38-42', unless there is raw data from which this can be observed and deduced. No such data was given. It is an incorrect means to coerce candidates into closing the polygon.

8. **Data:** A diagram showing three figures in a sequence.

(a) **Required to draw:** The fourth figure in the sequence.

Solution:

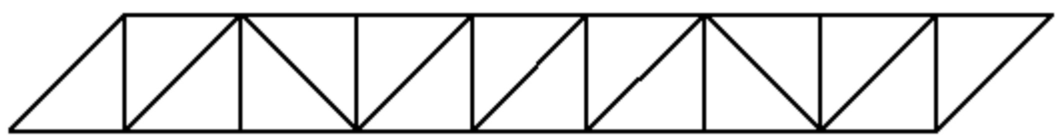
By observation, we notice that:

- Each figure is composed of squares drawn side by side and triangles at their ends.
- The number of squares in each new figure increases by 2. For example,
  Figure 1 has 1 square.
  Figure 2 has 1 + 2 = 3 squares.
  Figure 3 has 3 + 2 = 5 squares.
  So, by extrapolation, we would expect figure 4 to have 5 + 2 = 7 squares.

Another interesting observation made is that the end triangles in figure 1 and figure 3 are drawn the same way, but they are 'inverted' in figure 2. We would expect the end triangles of figure 4 to be drawn in the same way as that in figure 2.

The pattern may be also looked at in another way

We can also see that that figure 1 is 'flipped' and added (attached or concatenated) onto figure 1 to create figure 2. Then, figure 3 is obtained when figure 1 added to figure 2. So we realise that figure 4 is likely to be figure 1, first flipped and added (attached or concatenated) to figure 3.
Hence, this procedure has been carried out to obtain our fourth diagram in the sequence.
The fourth figure is expected to look like:



(b) **Required to complete:** The table given.
Solution:

No. of Trapezia (n)	No. of Triangles	No. of Dots
1	4	6
2	8	10
3	12	14

We notice that the number of triangles is always 4 times the number of trapezia, denoted by n
Check,

When $n=1$, the number of triangles $=4\times1=4$
When $n=2$, the number of triangles $=4\times2=8$
When $n=3$, the number of triangles $=4\times3=12$
Following this pattern to obtain the number of triangles we can say;

The number of triangles expected for n trapezia $= 4 \times n = 4n$

The number of dots is observed as always two more than the number of triangles.

Let us confirm this by the following observations:

When $n = 1$, number of triangles $= 4 \times 1 = 4$ and the number of dots = 4 + 2 = 6
When $n = 2$, number of triangles $= 4 \times 2 = 8$ and the number of dots= 8 + 2 = 10
When $n = 3$, number of triangles $= 4 \times 3 = 12$ and the number of dots=10 + 2=12

So if the number of trapezia = n
then, the number of triangles will be 4 times this number $= 4 \times n = 4n$
and the number of dots will be 2 more than the number of trapezia $= 4n + 2$
A row for any number of trapezia could look like:

n	$4n$	$4n+2$

We can now answer parts (i) to (iv), using the rules created.

	No. of Trapezia (n)	**No. of Triangles**	**No. of Dots**
	n	$4n$	$4n+2$
(i)	4	$4(4)=16$	$4(4)+2=16+2$ $=18$
(ii)	10	$4(10)=40$	$4(10)+2=40+2$ $=42$
(iii)	$\frac{64}{4}=16$	64	$64+2=66$
(iv)	n	$4n$	$4n+2$

Section II

9. (a) **Data:** $h(x)=\frac{10}{x}-3$ and $g(x)=3x-2$

(i) a) **Required to evaluate:** $g(4)$

Solution:

We substitute $x = 4$ in $g(x)$

$$g(x)=3x-2$$
$$g(4)=3(4)-2$$
$$=12-2$$
$$=10$$

b) **Required to evaluate:** $hg(4)$

Solution:

Using the fact that $g(4)$= 10 from (i), then

$$hg(4)\equiv h(10)$$
$$=\frac{10}{10}-3$$
$$=1-3$$
$$=-2$$

OR

Finding the composite function $hg(x)$ to get

$$hg(x)=\frac{10}{3x-2}-3$$
$$hg(4)=\frac{10}{3(4)-2}-3$$
$$=\frac{10}{10}-3$$
$$=-2$$

(ii) a) **Required to write:** An expression in terms of x for $h^{-1}(x)$.

Solution:

Let $y = h(x)$

Then make x the subject and finally replace y by x to obtain the inverse of the function.

$$y = \frac{10}{x} - 3$$

$$y + 3 = \frac{10}{x}$$

$$x(y+3) = 10$$

$$\therefore x = \frac{10}{y+3}$$

Replace y by x to obtain:

$h^{-1}(x) = \dfrac{10}{x+3}, \quad x \neq -3$ i.e the function is not valid for $x = 3$

b) **Required to write:** An expression in terms of x for $gg(x)$

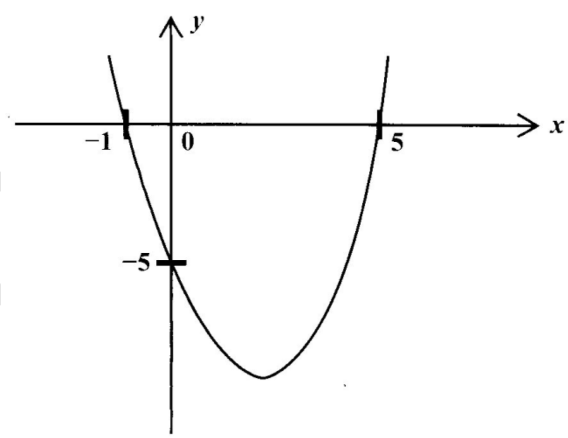
Solution:

We replace x in $g(x)$ by $g(x)$

$$\begin{aligned} gg(x) &= 3(3x-2)-2 \\ &= 9x-6-2 \\ &= 9x-8 \end{aligned}$$

(Note: $gg(x)$ may also have been written as $g^2(x)$)

(b) **Data:** Sketch of $y = x^2 + bx + c$ for $-2 \leq x \leq 6$, where b and c are constants.



(i) **Required to state:** The roots of the equation, $x^2 + bx + c = 0$.

Solution:

The curve $y = x^2 + bx + c$ cuts the horizontal axis at $x = -1$ and at $x = 5$.

$\therefore$ The roots are $x = -1$ and $x = 5$, the points at which the graph cuts the horizontal axis.

(ii) **a)** **Required to determine:** The value of c.

Solution:

If $x=-1$ and at $x=5$ are the roots of $y=x^2+bx+c$, then

$$x^2+bx+c=(x-(-1))(x-5)$$
$$=(x+1)(x-5)$$
$$=x^2-5x+x-5$$
$$=x^2-4x-5$$

Equating the constant coefficients, we obtain $c=-5$

OR

Let $y=x^2+bx+c$

The curve cuts the vertical axis when $x = 0$

When $x=0$

$$y=(0)^2+b(0)+c$$
$$=c$$

$\therefore$ The curve cuts the y – axis at c and where $c=-5$ as seen on the diagram.

b) **Required to prove:** $b=-4$

Proof:

Recall that

$$x^2+bx+c=(x+1)(x-5)$$
$$=x^2+x-5x-5$$
$$=x^2-4x-5$$

Equating the constant term of the equation to obtain

$b=-4$

Q.E.D.

(iii) **Required to state:** The coordinates of the minimum point on the graph of the function, $y=x^2+bx+c$.

Solution:

$y=x^2+bx+c$, was found to be

$y=x^2-4x-5$

Hence, the minimum value of y occurs at the x-coordinate, the same x value as the equation of axis of symmetry. So, the minimum value occurs at

$$x=\frac{-(-4)}{2(1)}$$
$$=2$$

When $x=-2$

$y=(2)^2-4(2)-5$

$=-9$

The minimum value is -9

$\therefore$ The minimum point has coordinates $(2,\,-9)$.

OR

The roots are at $x = -1$ and $x = 5$. The minimum point occurs when x is 'halfway' between the roots, that is at $x=\frac{-1+5}{2}$

$=2$

When $x = 2$

$y=(2)^2-4(2)-5$

$=-9$

$\therefore$ The minimum point has coordinates $(2,\,-9)$.

OR

x^2-4x-5

We are using the method of completing the squares.

Half the coefficient of x is $\frac{1}{2}(-4)=-2$

$x^2-4x-5=(x-2)^2+?$ and where the value of ? has to be found.

$$\begin{aligned}(x-2)^2 &= x^2-4x+4\\ &\qquad\quad \underline{-9} = ?\\ &\qquad\quad \underline{-5}\end{aligned}$$

$\therefore x^2-4x-5=(x-2)^2-9$ (obtained by the completion of the square)

$\therefore$ The minimum value of occurs when $(x-2)^2 = 0$

(any quantity to be squared is greater than or equal to zero)

The minimum value of $f(x)$ can be found letting $(x-2)^2 = 0$

$f(x)=(x-2)^2-9$

$=0^2-9$

$=-9$

When $(x-2)^2=0$

$x=2$.

$\therefore$ The minimum point is $(2,\,-9)$.

OR

Advanced students of mathematics may even use differential calculus, since the question did not specify any particular method.

Let $y = x^2 - 4x - 5$

The gradient function, $\frac{dy}{dx} = 2x - 4$

A stationary point occurs when, the gradient function, $\frac{dy}{dx} = 0$.

So, we let

$2x - 4 = 0$

$x = 2$

When $x = 2$

$y = (2)^2 - 4(2) - 5$

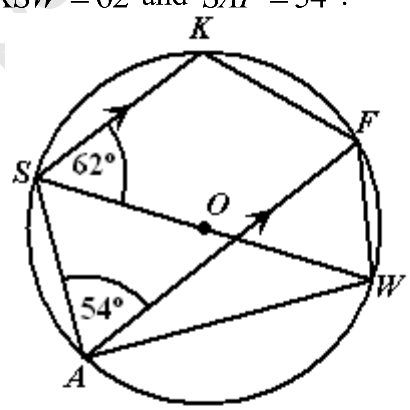
$= -9$

$\therefore$ The stationary point is $(2, -9)$.

The second derivative $\frac{d^2y}{dx^2} = 2 > 0$ and which confirms that the point $(2, -9)$ is a minimum point.

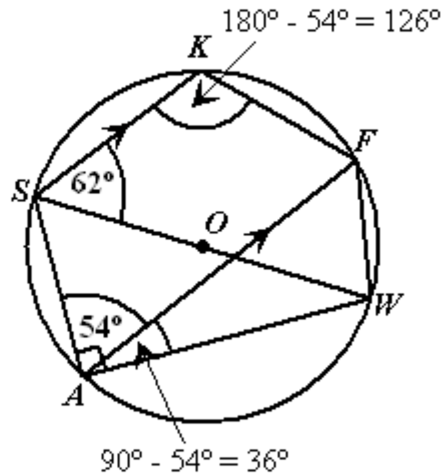
10. (a) Data: Circle with center O.

SK and AF are parallel. $K\hat{S}W = 62°$ and $S\hat{A}F = 54°$.



(i) **Required to calculate:** $F\hat{A}W$

Calculation:



Consider the triangle SAW

$S\hat{A}W = 90°$ (the angle in a semi-circle is 90°).

$\therefore F\hat{A}W = 90° - 54°$

$= 36°$

(ii) **Required to calculate:** The size of $S\hat{K}F$

Calculation:

Consider the cyclic quadrilateral $SKFA$ shown in the diagram.

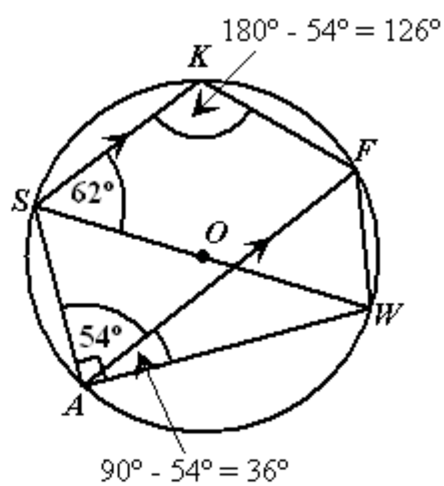
$S\hat{K}F = 180° - 54°$

$= 126°$

(The opposite angles of a cyclic quadrilateral are supplementary)

(iii) **Required to calculate:** $A\hat{S}W$

Calculation:



Let us consider the quadrilateral $KSAF$

The opposite sides SK and AF are parallel

The angles KSA and SAF are co-interior opposite angles and are therefore supplementary.

(When parallel lines are cut by a transversal, co-interior opposite angles are supplementary)

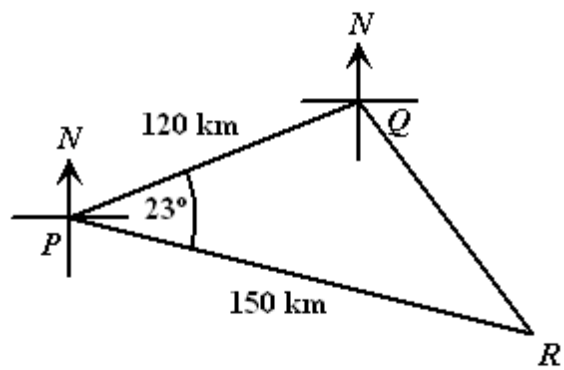
Therefore $54^0 + 62^0 + \text{angle } ASW = 180^0$

So $A\hat{S}W = 64°$

(b) **Data:** A diagram showing the points P, Q and R.

(i) **a)** **Required to calculate:** The distance QR

Calculation:



Applying the cosine rule to triangle PQR

$$QR^2 = (120)^2 + (150)^2 - 2(120)(150)\cos 23°$$
$$= 14\,400 + 22\,500 - 36\,000 \cos 23°$$
$$= 36\,900 - 33138.17$$
$$= 3\,761.83$$
$$QR = 61.3\underline{\underline{3}}$$
$$= 61.3 \text{ km (correct to 1 decimal place)}$$

b) **Required to calculate:** The area of triangle PQR.

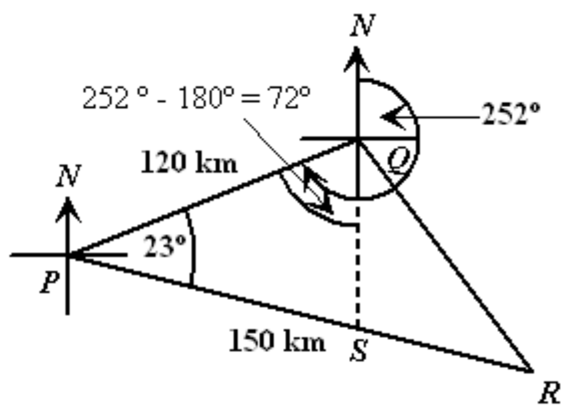
Calculation:

The area of triangle $PQR = \frac{1}{2}\text{side} \times \text{side} \times \sin(\text{included angle})$

We have the sides PQ and PR and the included angle

$$= \frac{1}{2}(120)(150)\sin 23°$$
$$= 3516.5\underline{\underline{8}}$$
$$= 3516.6 \text{ km}^2 \text{ (correct to 1 decimal place)}$$

(ii) **Data:** The bearing of *P* from *Q* is 252°.
Required to calculate: The bearing of *R* from *P*
Calculation:



Let the South line from *Q* meet *PR* at *S*, as shown on the above diagram
The line *NS* is a straight line and the angle *NQS* is 180^0

$$P\hat{Q}S = 252° - 180°$$
$$= 72°$$

The angle *NPQ* = 72^0
(Alternate angle to angle *PQS*)

$$N\hat{P}R = 72° + 23° = 95°$$

Therefore the bearing of *R* from *P* is
$$= 095° \text{ (bearing must have 3 digits)}$$

11. (a) **Data:** $T = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$

(i) **Required to determine:** T^{-1}
Solution:
First we find the determinant of *T*

$$\det T = (2\times3) - (-1\times1)$$
$$= 6 - (-1)$$
$$= 7$$

$$T^{-1} = \frac{1}{7}\begin{pmatrix} 3 & -(-1) \\ -(1) & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{7} & \frac{1}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{pmatrix}$$

(ii) **Data:** *T* maps (a, b) onto $(4, 9)$.

Required to determine: The value of *a* and of *b*.

Solution:

$$\begin{pmatrix} a \\ b \end{pmatrix} \xrightarrow{T} \begin{pmatrix} 4 \\ 9 \end{pmatrix}$$

$$\xleftarrow{T^{-1}}$$

If (a, b) is mapped onto $(4, 9)$ under *T*, then $(4, 9)$ will be mapped onto (a, b) under T^{-1}. So we multiply the point (4,9) by the inverse of *T* to get the value of *a* and of *b*.

$$\therefore \begin{pmatrix} \frac{3}{7} & \frac{1}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{pmatrix}\begin{pmatrix} 4 \\ 9 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{3}{7}\times 4 & \frac{1}{7}\times 9 \\ -\frac{1}{7}\times 4 & \frac{2}{7}\times 9 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{21}{7} \\ \frac{14}{7} \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

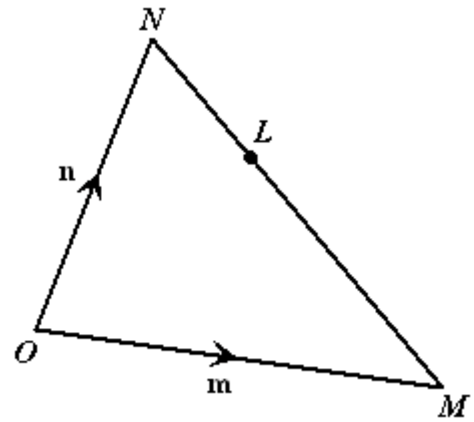
Equating corresponding entries since both sides are 2 x1 equal matrices. We equate them to obtain *a* and *b*.

$\therefore a = 3$ and $b = 2$

(b) **Data:** $\overrightarrow{OM} = \mathbf{m}$ and $\overrightarrow{ON} = \mathbf{n}$. *L* is on *MN* such that *ML* : *LN* = 2:1.

(i) **Required to sketch**: Triangle *OMN*.

Solution:



(ii) **a)** **Required to write:** $\overrightarrow{MN}$ in terms of **m** and **n**.
Solution:
Applying the vector triangle law to triangle MON

$$\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON}$$
$$= -(\mathbf{m}) + \mathbf{n}$$
$$= -\mathbf{m} + \mathbf{n} \text{ or } \mathbf{n} - \mathbf{m}$$

b) **Required to write:** $\overrightarrow{ML}$ in terms of **m** and **n**.
Solution:

$$ML = \frac{2}{2+1} \text{ of } MN$$
$$\overrightarrow{ML} = \left(\frac{2}{2+1}\right) \times \overrightarrow{MN}$$
$$= \frac{2}{3}(-\mathbf{m} + \mathbf{n})$$
$$= -\frac{2}{3}\mathbf{m} + \frac{2}{3}\mathbf{n}$$

(iii) **Data:** $\mathbf{m} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ and $\mathbf{n} = \begin{pmatrix} 9 \\ 0 \end{pmatrix}$

Required to determine: The position vector of L
Solution:
The position vector of L is $\overrightarrow{OL}$, the vector being measured with respect to the origin, O.
Applying the vector triangle law.

$$
\begin{aligned}
OL &= OM + ML \\
&= \mathbf{m} + \left(-\frac{2}{3}\mathbf{m} + \frac{2}{3}\mathbf{n}\right) \\
&= \frac{1}{3}\mathbf{m} + \frac{2}{3}\mathbf{n} \\
&= \frac{1}{3}\begin{pmatrix}3\\6\end{pmatrix} + \frac{2}{3}\begin{pmatrix}9\\0\end{pmatrix} \\
&= \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}6\\0\end{pmatrix} \\
&= \begin{pmatrix}7\\2\end{pmatrix}
\end{aligned}
$$

$\therefore$ The position vector of L is $\begin{pmatrix}7\\2\end{pmatrix}$.