JANUARY 2007 MATHEMATICS GENERAL PROFICIENCY (PAPER 2)

Section I

1. a. (i) **Required To Calculate:** $5.24(4-1.67)$

Calculation:

$$5.24(4-1.67) = 5.24(2.33)$$
$$= 12.2\underline{0}92 \quad \text{(exactly)}$$
$$= 12.2 \textit{ to } 1 \text{ decimal place}$$

(ii) **Required To Calculate:** $\dfrac{1.68}{1.5^2 - 1.45}$

Calculation:

$$\frac{1.68}{1.5^2 - 1.45} = \frac{1.68}{2.25 - 1.45}$$
$$= \frac{1.68}{0.8}$$
$$= 2.1 \text{ (exactly)}$$

b. **Data:** Aaron received 2 shares totaling \$60 from a sum shared in the ratio $2:5$.
Required To Calculate: The sum of money.
Calculation:
Aaron's 2 shares total \$60

$$\therefore 1 \text{ share} \equiv \frac{\$60}{2}$$
$$= \$30$$

Total no. of shares = 2 + 5 = 7
Sum that was shared altogether

$$= \$30 \times 7$$
$$= \$210$$

c. **Data:** Cost of gasoline is \$10.40 for 3 litres. All currency in \$EC.

(i) **Required To Calculate:** Cost of 5 litres of gasoline.
Calculation:
If 3 litres of gasoline cost \$10.40

Then 1 litre of gasoline costs $\dfrac{\$10.40}{3}$

And 5 litres of gasoline cost $\dfrac{\$10.40}{3} \times 5$

$$= \$17.33\underline{3}$$
$$= \$17.33 \text{ to nearest cent}$$

(ii) **Required To Calculate:** Volume of gasoline that can be bought with \$50.00.

Calculation:

\$10.40 affords 3 litres

\$1.00 will afford $\frac{3}{10.40}$ litres

\$50.00 will afford $\left(\frac{3}{10.40} \times 50.00\right)$ litres

$= 14.4$ litres

$= 14$ litres to the nearest whole number

2. a. **Data:** $a = 2$, $b = -3$ and $c = 4$

(i) **Required To Calculate:** $ab - bc$

Calculation:

$$ab - bc = 2(-3) - (-3)4$$
$$= -6 + 12$$
$$= 6$$

(ii) **Required To Calculate:** $b(a-c)^2$

Calculation:

$$b(a-c)^2 = -3(2-4)^2$$
$$= -3(-2)^2$$
$$= -3(4)$$
$$= -12$$

b. (i) **Data:** $\frac{x}{2} + \frac{x}{3} = 5$

Required To Find: x where $x \in Z$

Solution:

$$\frac{x}{2} + \frac{x}{3} = \frac{5}{1}$$

$\times 6$

$$6\left(\frac{x}{2}\right) + 6\left(\frac{x}{3}\right) = 6\left(\frac{5}{1}\right)$$
$$3x + 2x = 30$$
$$5x = 30$$
$$x = 6 \in Z$$

OR

$$\frac{x}{2}+\frac{x}{3}=5$$
$$\frac{3(x)+2(x)}{6}=5$$
$$\frac{5x}{6}=5$$
$\times 6$
$$5x=30$$
$$x=6\in Z$$

(ii) **Data:** $4-x\leq 13$
Required To Find: x where $x\in Z$
Solution:
$4-x\leq 13$
$-x\leq 13-4$
$\times -1$
$x\geq -9$
That is $x=\{-9,-8,-7,\ldots,x\in Z\}$

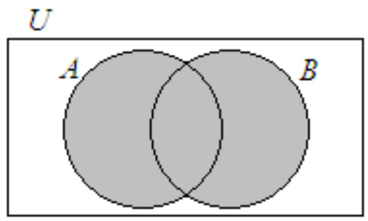
c. **Data:** 1 muffin costs $\$m$ and 3 cupcakes cost $\$2m$

(i) (a) **Required To Find:** Cost of five muffins in terms of *m*.
Solution:
1 muffin costs $\$m$
5 muffins costs $\$(m\times 5)$
$=\$5m$

(b) **Required To Find:** Cost of six cupcakes in terms of *m*.
Solution:
If 3 cupcakes cost $\$2m$
Then 1 cupcake costs $\$\frac{2m}{3}$
And 6 cupcakes cost $\$\frac{2m}{3}\times 6$
$=\$4m$

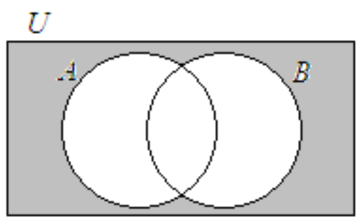
(ii) **Required To Find:** An equation for the total cost of 5 muffins and 6 cupcakes is $31.50.
Solution:
$\$5m+\ \$4m=\$9m$
Hence, $\$9m=\$31.50$
$9m=31.50$

3. a. (i) **Data:**



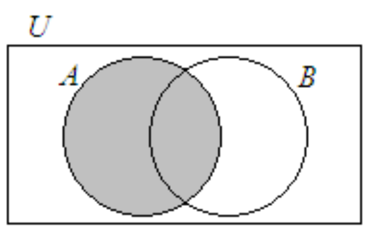
Required To Describe: The shaded region using set notation.
Solution:
The region shaded is all of sets A and B, that is $A \cup B$.

(ii) **Data:**



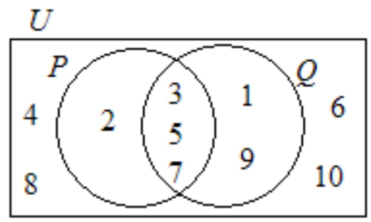
Required To Describe: The shaded region using set notation.
Solution:
The region shaded is the region in U except $A \cup B$, that is $(A \cup B)'$.

(iii) **Data:**

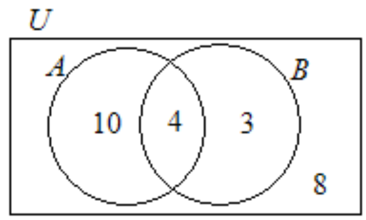


Required To Describe: The shaded region using set notation.
Solution:
The region shaded in the set A only. Hence the shaded region is A.

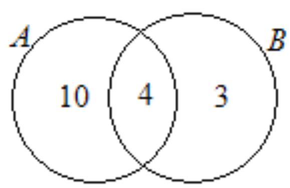
b. **Data:** $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$P = \{\text{Prime numbers}\}$
$Q = \{\text{Odd numbers}\}$
Required To Draw: A Venn diagram to represent the information given.
Solution:
$P = \{2, 3, 5, 7\}$
$Q = \{1, 3, 5, 7, 9\}$



c. **Data:** Venn diagram illustrating the number of elements in each region.

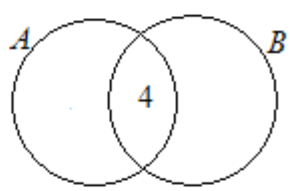


(i) **Required To Find:** No. of elements in $A \cup B$.
Solution:



$$n(A \cup B) = 10 + 4 + 3$$
$$= 17$$

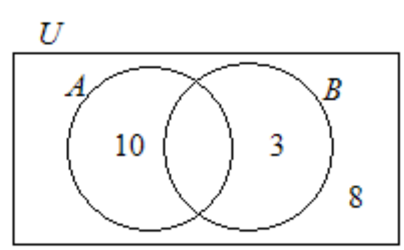
(ii) **Required To Find:** No. of elements in $A \cap B$.
Solution:



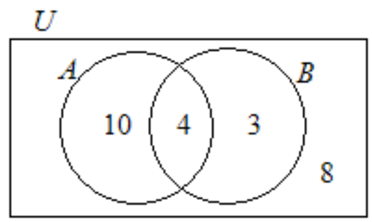
$$n(A \cap B) = 4$$

(iii) **Required To Find:** No of elements in $(A \cap B)'$.
Solution:

$$n(A \cap B)' = 10 + 3 + 8$$
$$= 21$$

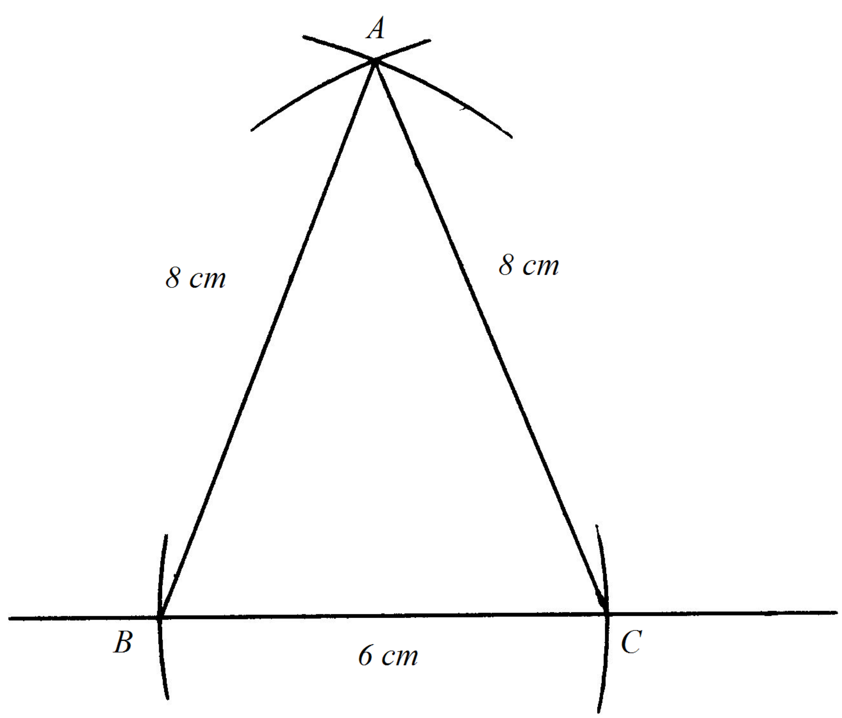


(iv) **Required To Find:** No. of elements in U.
Solution:



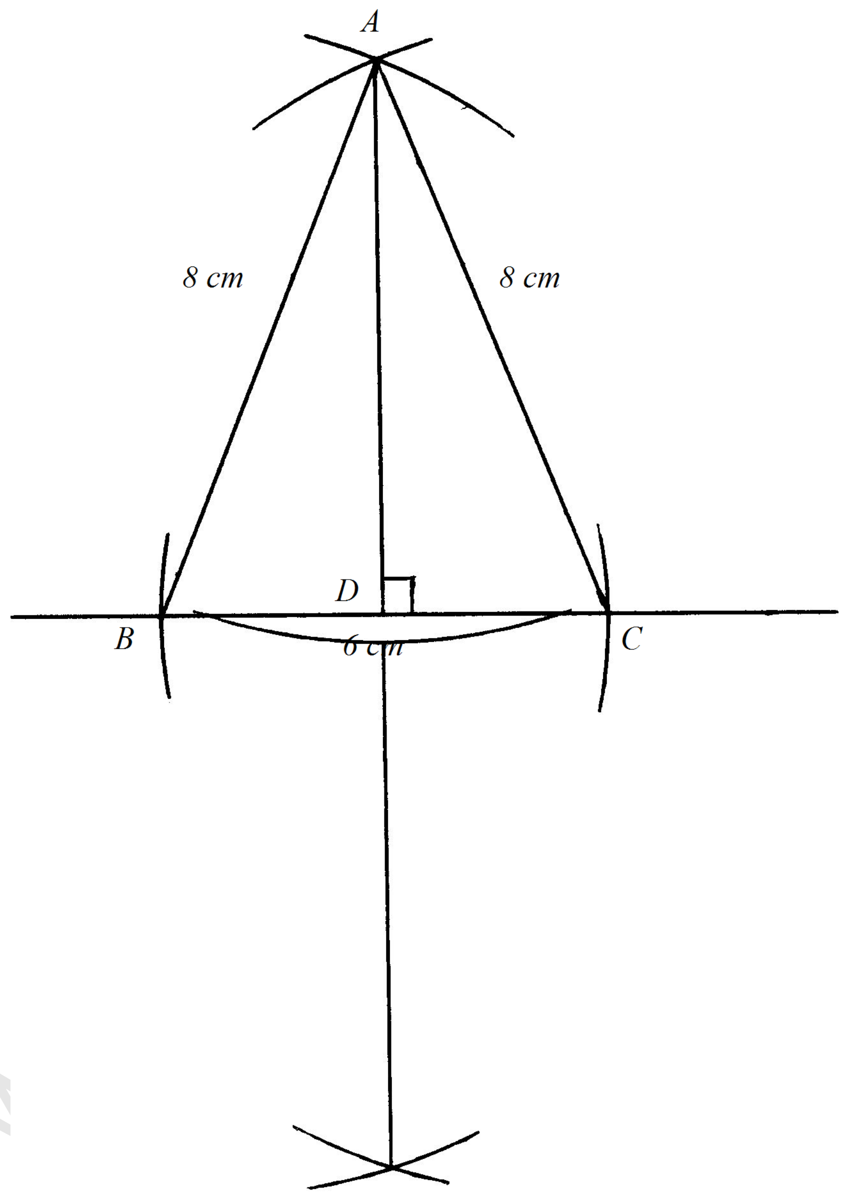
$$n(U) = 10 + 4 + 3 + 8$$
$$= 25$$

4. a. (i) **Required To Construct:** ΔABC with $BC = 6\text{cm}$ and $AB = AC = 8\text{cm}$.
Solution:



(ii) **Required To Construct:** AD such that AD meets BC at D and is perpendicular to BC.

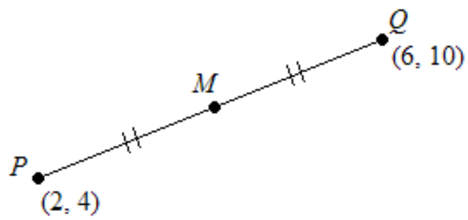
Solution:



(iii) (a) **Required To Find:** Length of AD.
Solution:
$AD = 7.3$ cm (by measurement)

(b) **Required To Find:** Size of $A\hat{B}C$
Solution:
$A\hat{B}C = 68°$ (by measurement)

b. **Data:** $P = (2,\ 4)$ and $Q = (6,\ 10)$



(i) **Required To Calculate:** Gradient of PQ.
Calculation:

$$\text{Gradient of } PQ = \frac{10-4}{6-2} = \frac{6}{4} = \frac{3}{2}$$

(ii) **Required To Calculate:** Midpoint of PQ.
Calculation:
Let midpoint of PQ be M.

$$M = \left(\frac{2+6}{2}, \frac{4+10}{2}\right) = (4, 7)$$

5. a. **Data:** $f(x) \to 7x+4$ and $g(x) \to \frac{1}{2x}$

(i) **Required To Calculate:** $g(3)$
Calculation:

$$g(3) = \frac{1}{2(3)} = \frac{1}{6}$$

(ii) **Required To Calculate:** $f(-2)$
Calculation:

$$f(-2) = 7(-2)+4 = -14+4 = -10$$

(iii) **Required To Calculate:** $f^{-1}(11)$

Calculation:

Let $y = 7x + 4$

$y - 4 = 7x$

$\frac{y-4}{7} = x$

Replace y by x

$\therefore f^{-1}(x) = \frac{x-4}{7}$

$\therefore f^{-1}(11) = \frac{11-4}{7}$

$= \frac{7}{7}$

$= 1$

b. (i) $x = 5$

$A'' = (1,2)$

(ii) $B'' = (3,2)$

$C'' = (3,-1)$

(iii) Reflection in the line $y = 4$

6. **Data:** Table showing a frequency distribution of scores of 100 students in an examination.

(i) **Required To Complete:** And modify the table given.

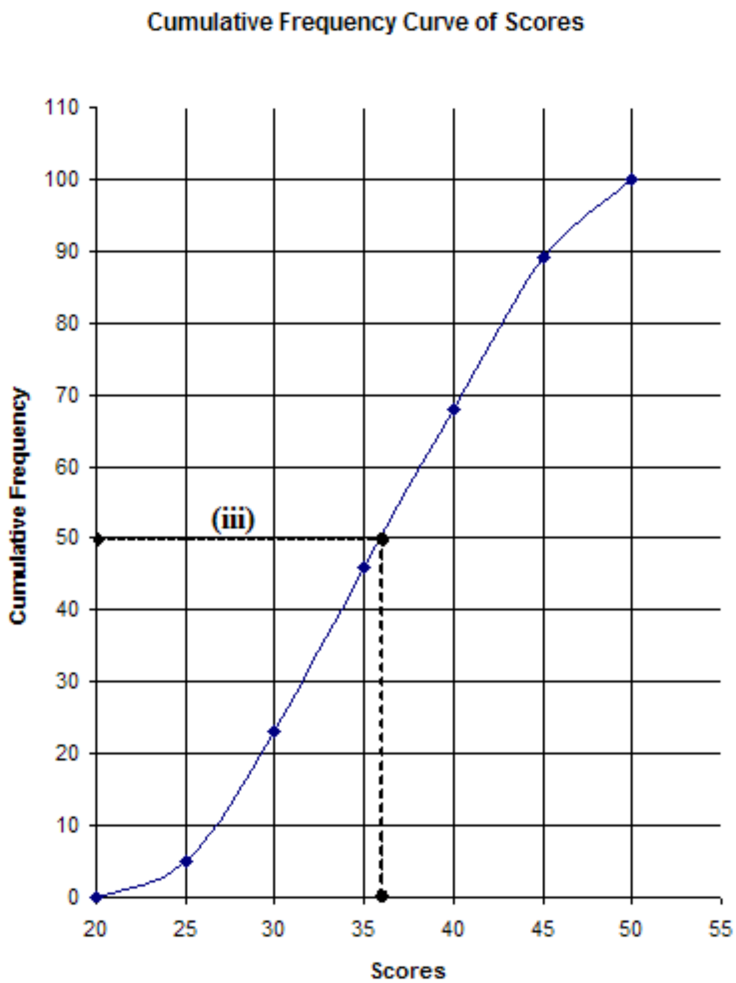
Solution:

Score (Discrete Variable)	U.C.B	Frequency	Cumulative Frequency	Points to Plot (U.C.B, C.F.)
				(20, 0)
21 – 25	25	5	5	(25, 5)
26 – 30	30	18	$18 + 5 = 23$	(30, 23)
31 – 35	35	23	$23 + 23 = 46$	(35, 46)
36 – 40	40	22	$22 + 46 = 68$	(40, 68)
41 – 45	45	21	$21 + 68 = 89$	(45, 89)
46 – 50	50	11	$11 + 89 = 100$	(50, 100)

$\sum f = 100$

The point (20, 0) corresponding to an upper class boundary of 20 and a cumulative frequency value of 0, obtained by checking ‘backwards’, is to be plotted, as the graph of cumulative frequency starts from the horizontal axis.

(ii) **Data:** Scale is 2 cm to represent 5 units on the horizontal axis and 2 cm to represent 10 units on the vertical axis.
Required To Plot: The cumulative frequency curve of the scores.
Solution:



(iii) **Required To Find:** Median score.
Solution:
From the cumulative frequency curve, the median score corresponds to a cumulative frequency value of $\frac{1}{2}(100) = 50$ and reads as 36 on the horizontal axis.
$\therefore$ Median score = 36.

(iv) **Required To Calculate:** Probability a randomly chosen student has a score greater than 40.
Solution:

$$P(\text{student chosen at random scores} > 40) = \frac{\text{No. of students scoring} > 40}{\text{Total no. of students}}$$

$$= \frac{21+11}{\sum f = 100}$$

$$= \frac{32}{100}$$

$$= \frac{8}{25}$$

7. a. **Data:** Prism of cross-sectional area 144 cm^2 and length 30 cm.

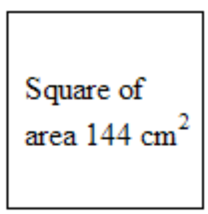
(i) **Required To Calculate:** Volume of the prism.
Calculation:

Volume of prism = Area of cross-section $\times$ Length

$$= 144 \times 30 \text{ cm}^3$$

$$= 4320 \text{ cm}^3$$

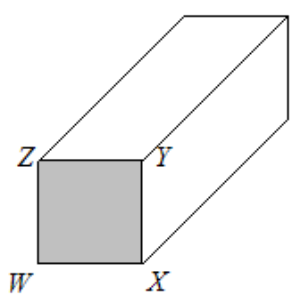
(ii) **Required To Calculate:** Total surface area of the prism.
Calculation:



Cross-section is a square of area 144 cm^2.

$$\therefore \text{Length} = \sqrt{144 \text{ cm}^2}$$

$$= 12 \text{ cm}$$



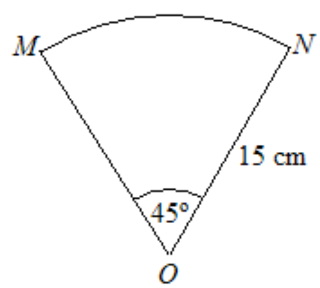
Area of front and back faces $= 144 \times 2$
$= 288 \text{ cm}^2$

Area of L.H.S and R.H.S. rectangular faces $= 2(12 \times 30)$
$= 720 \text{ cm}^2$

Area of top and base rectangular faces $= 2(12 \times 30)$
$= 720 \text{ cm}^2$

Total surface area of the prism $= 288 + 720 + 720$
$= 1728 \text{ cm}^2$

b. **Data:**



MON is a sector of a circle of radius 15 cm and $M\hat{O}N = 45°$.

(i) **Required To Calculate:** Length of minor arc *MN*.
Calculation:

Length of arc $MN = \dfrac{45}{360} \times 2\pi(15)$
$= 11.77\underline{5}$
$= 11.78 \text{ cm to 2 decimal places}$

OR

Using
$s = r\theta$
s = arc length, r = radius and θ = angle in radians
$s = (15)(0.785) = 11.775$
$s = 11.78 \text{ cm to 2 decimal places}$

(ii) **Required To Calculate:** Perimeter of figure *MON*.
Calculation:
Perimeter of *MON* = Arc length *MON* + Length of radius *OM* + Length of radius *ON*

$$= 11.775 + 15 + 15$$
$$= 41.775$$
$$= 41.78 \text{ to 2 decimal places}$$

(iii) **Required To Calculate:** Area of figure *MON*.
Calculation:

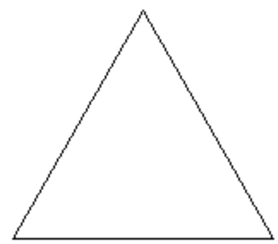
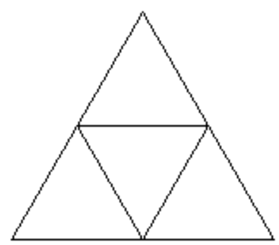
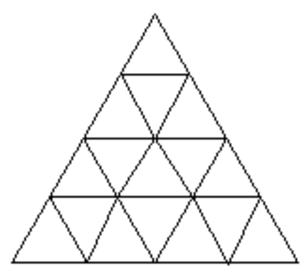
$$\text{Area of sector } MON = \frac{45}{360} \times \pi(15)^2$$
$$= 88.31\underline{2}$$
$$= 88.31\,\text{cm}^2 \text{ to 2 decimal places}$$

OR

$$\text{Area of sector } = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2}(15)^2(0.785)$$
$$= 88.31\underline{2}$$
$$= 88.31\text{ cm}^2 \text{ to 2 decimal places}$$

8. **Data:** Table showing the subdivision of an equilateral triangle.
Required To Complete: The table given.

Solution:

n	**Result of each step**	**No. of triangles formed**
0		$1 = 4^0$
1		$4 = 4^1$
2		$16 = 4^2$
3		(i) $64 = 4^3$
⋮	⋮	⋮
6		(ii) $4096 = 4^6$
⋮	⋮	⋮
(iii) 8		$65536 = 4^8$
⋮	⋮	⋮
m		(iv) 4^m

$$\begin{array}{r|l} 4 & \underline{65536} \\ 4 & \underline{16384} \\ 4 & \underline{4096} \\ 4 & \underline{1024} \\ 4 & \underline{256} \\ 4 & \underline{64} \\ 4 & \underline{16} \\ 4 & \underline{4} \\ & \underline{1} \end{array}$$

Section II

9. a. **Required To Factorise:** (i) $2p^2 - 7p + 3$, (ii) $5p + 5q + p^2 - q^2$

Factorising:

(i) $2p^2 - 7p + 3$

$= (2p-1)(p-3)$

(ii) $5p + 5q + p^2 - q^2$

$= 5(p+q) + (p-q)(p+q)$

$= (p+q)\{5 + (p-q)\}$

$= (p+q)(5+p-q)$

b. **Required To Expand:** $(x+3)^2(x-4)$

Solution:

Expanding

$$
\begin{aligned}
(x+3)^2(x-4) &= (x+3)(x+3)(x-4) \\
&= (x^2 + 3x + 3x + 9)(x-4) \\
&= (x^2 + 6x + 9)(x-4) \\
&= x^3 + 6x^2 + 9x - 4x^2 - 24x - 36 \\
&= x^3 + 2x^2 - 15x - 36
\end{aligned}
$$

Hence, $(x+3)^2(x-4) = x^3 + 2x^2 - 15x - 36$, in descending powers of x.

c. **Data:** $f(x) = 2x^2 + 4x - 5$

(i) **Required To Express:** $f(x) = 2x^2 + 4x - 5$ in the form $a(x+b)^2 + c$.

Solution:

$$
\begin{aligned}
f(x) &= 2x^2 + 4x - 5 \\
&= 2(x^2 + 2x) - 5
\end{aligned}
$$

(Half the coefficient of x is $\frac{1}{2}(2) = 1$)

Hence $f(x) = 2x^2 + 4x - 5$

$= 2(x+1)^2 + *$

$= 2(x^2 + 2x + 1) + *$

$= 2x^2 + 4x + 2$ (Hence * = -7)

$\underline{-7}$

$\underline{-5}$

$\therefore 2x^2 + 4x - 5 \equiv 2(x+1)^2 - 7$ is of the form $a(x+b)^2 + c$ where

$a = 2 \in \Re$

$b = 1 \in \Re$

$c = -7 \in \Re$

OR

$$\begin{aligned} 2x^2 + 4x - 5 &= a(x+5)^2 + c \\ &= a(x^2 + 2bx + b^2) + c \\ &= ax^2 + 2abx + ab^2 + c \end{aligned}$$

Equating coefficient of x^2.

$a = 2 \in \Re$

Equating coefficient of x.

$2(2)b = 4$

$b = 1 \in \Re$

Equating constants.

$2(1)^2 + c = -5$

$c = -7 \in \Re$

$\therefore 2x^2 + 4x - 5 \equiv 2(x+1)^2 - 7$

(ii) **Required To Find:** The equation of the axis of symmetry.

Solution:

If $y = ax^2 + bx + c$ is any quadratic curve, the axis of symmetry has equation $x = \dfrac{-b}{2a}$.

The equation of the axis of symmetry in the quadratic curve $f(x) = 2x^2 + 4x - 5$ is $x = \dfrac{-(4)}{2(2)}$

$x = -1$

(iii) **Required To Find:** Coordinates of the minimum point on the curve.

Solution:

$$\begin{aligned} f(x) &= 2x^2 + 4x - 5 \\ &= 2(x+1)^2 - 7 \end{aligned}$$

$2(x+1)^2 \geq 0 \quad \forall x$

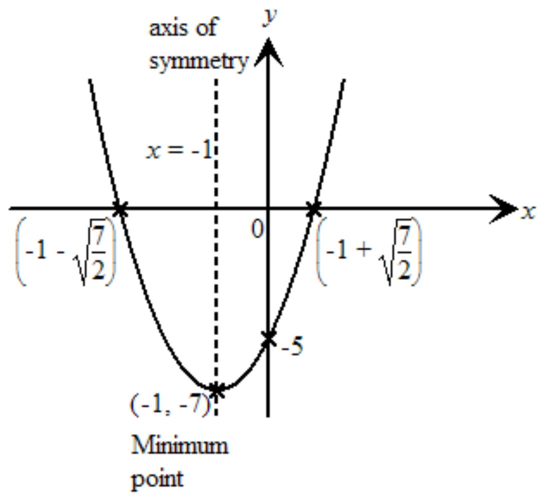
$\therefore f(x)_{\min} = -7$ at $2(x+1)^2 = 0$

$x = -1$

(iv) – (v)

Required To Draw: The graph of $f(x)$ showing the minimum point and the axis of symmetry.

Solution:



10. **Data:** Pam must buy x pens and y pencils.

a. (i) **Data:** Pam must buy at least 3 pens.

Required To Find: An inequality to represent the above information.

Solution:

No. of pens bought $= x$

No. of pens is at least 3.

$\therefore x \geq 3$

(ii) **Data:** Total number of pens and pencils must not be more than 10.

Required To Find: An inequality to represent the above information.

Solution:

No. of pencils $= y$

Total number of pens and pencils $= x + y$

$\therefore (x + y)$ is not more than 10.

$\therefore x + y$ is less than or equal to 10.

$x + y \leq 10$

(iii) **Data:** $5x + 2y \leq 35$

Required To Find: Information represented by this inequality.

Solution:

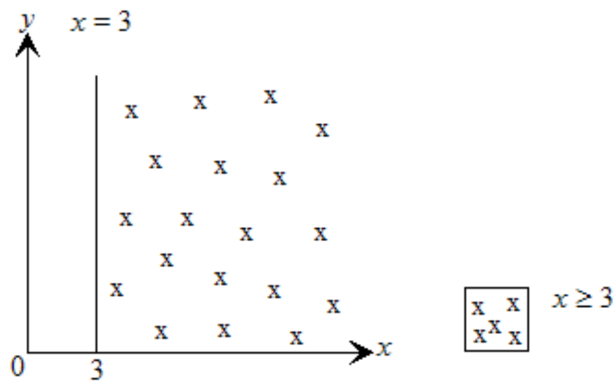
$5x + 2y \leq 35$ (data)

$5x$ represents the cost of x pens at \$5.00 each and $2y$ represents the cost of y pencils at \$2.00 each.

Total cost is $5x + 2y$.

Since $5x+2y\le 35$, then the total cost of x pens and y pencils is less than or equal to \$35.00
That is, the total cost of the x pens and y pencils is not more than \$35.00.

b. (i) **Required To Draw:** The graphs of the two inequalities obtained on answer sheet.
Solution:
The line $x=3$ is a vertical line.
The region $x\ge 3$ is



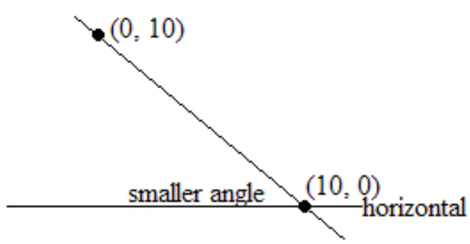
Obtaining 2 points on the line $x+y=10$

When $x=0$ $\quad 0+y=10$
$y=10$

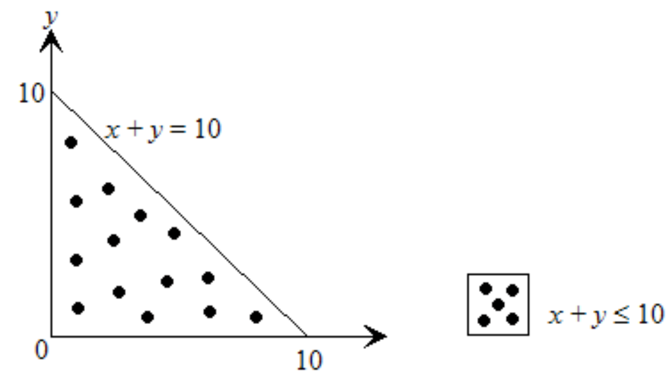
The line $x+y=10$ passes through the point (0, 10).

When $y=0$ $\quad x+0=10$
$x=10$

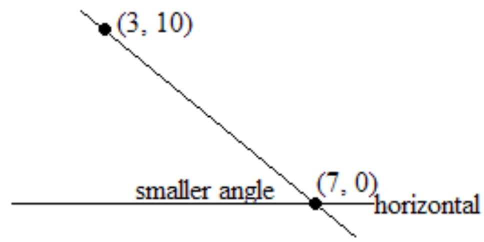
The line $x+y=10$ passes through the point (10, 0).



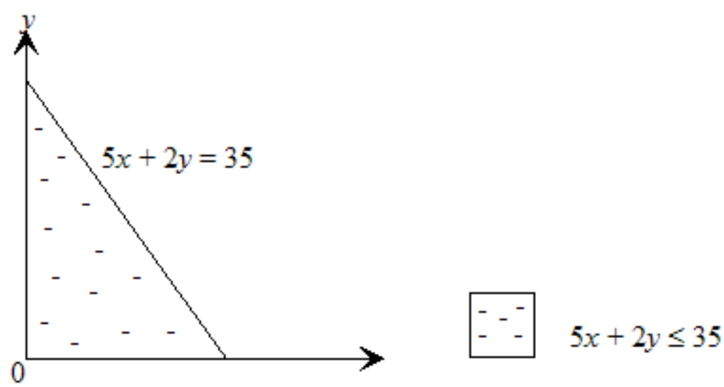
The region with the smaller angle satisfies the $\le$ region.
The region with satisfies $x+y\le 10$ is



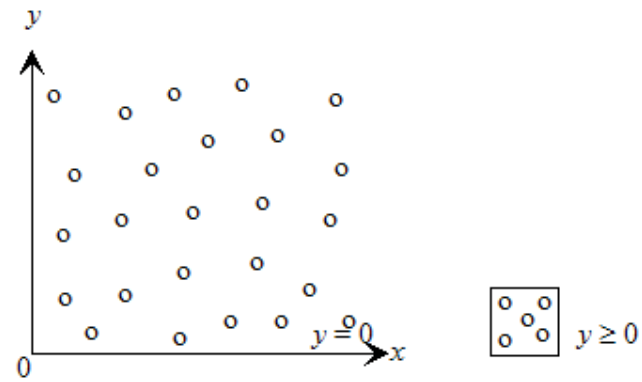
The graph of the line $5x + 2y = 35$ was given. It passes through the points (7, 0) and (3, 10).



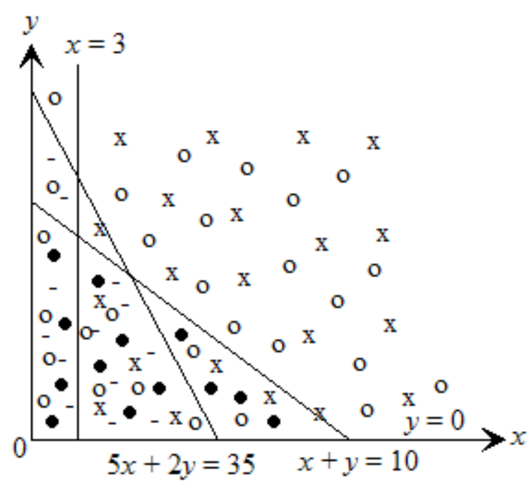
The region with the smaller angle satisfies the $\le$ region.
The region which satisfies $5x + 2y \le 35$ is

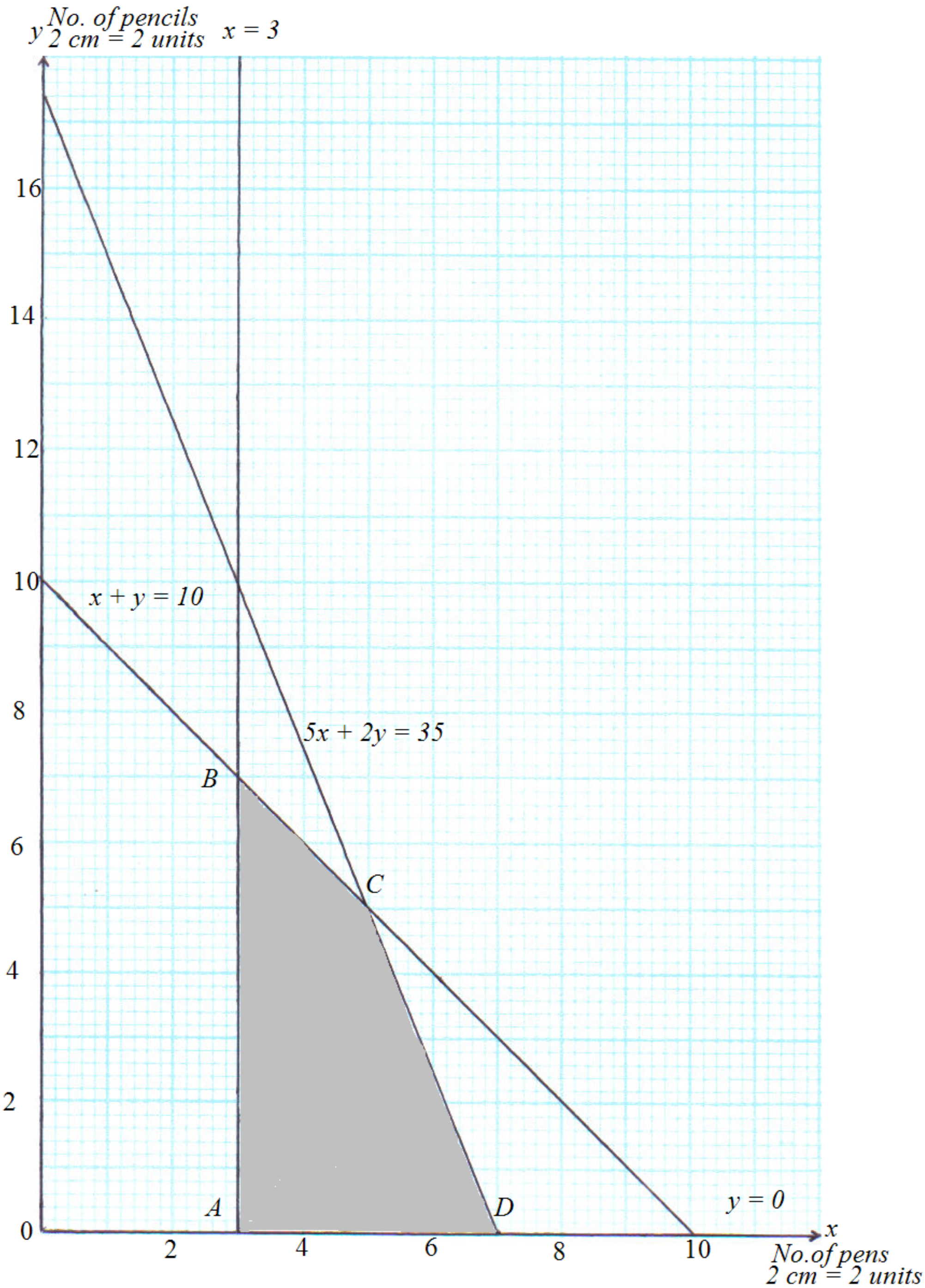


The line $y = 0$ is the horizontal x – axis.
The region which satisfies $y \ge 0$ is



The region which satisfies all four inequalities is the area in which all four previously shaded regions overlap. The region which satisfies all four inequalities is





(ii) **Required To Find:** The vertices of the region bounded by the 4 inequalities is shown ABCD (the feasible region)

Solution:

$A\ (3, 0)$ $B\ (3, 7)$ $C\ (5, 5)$ $D\ (7, 0)$

c. **Data:** A profit of \$1.50 is made on each pen and a profit of \$1.00 is made on each pencil.

(i) **Required To Find:** The profit in terms of x and y.
Solution:
Let the total profit on pens and pencils be P. The profit on x pens at $\$1\frac{1}{2}$ and y pencils at \$1 each $=\left(x\times 1\frac{1}{2}\right)+(y\times 1)$

$$\therefore P = 1\frac{1}{2}x + y$$

(ii) **Required To Find:** Maximum profit.
Solution:
Choosing only B (3, 7), C (5, 5) and D (7, 0).
At B $x = 3$ $y = 7$

$$P = 3\left(1\frac{1}{2}\right) + 7$$
$$= \$11\frac{1}{2}$$
$$= \$11.50$$

At C $x = 5$ $y = 5$

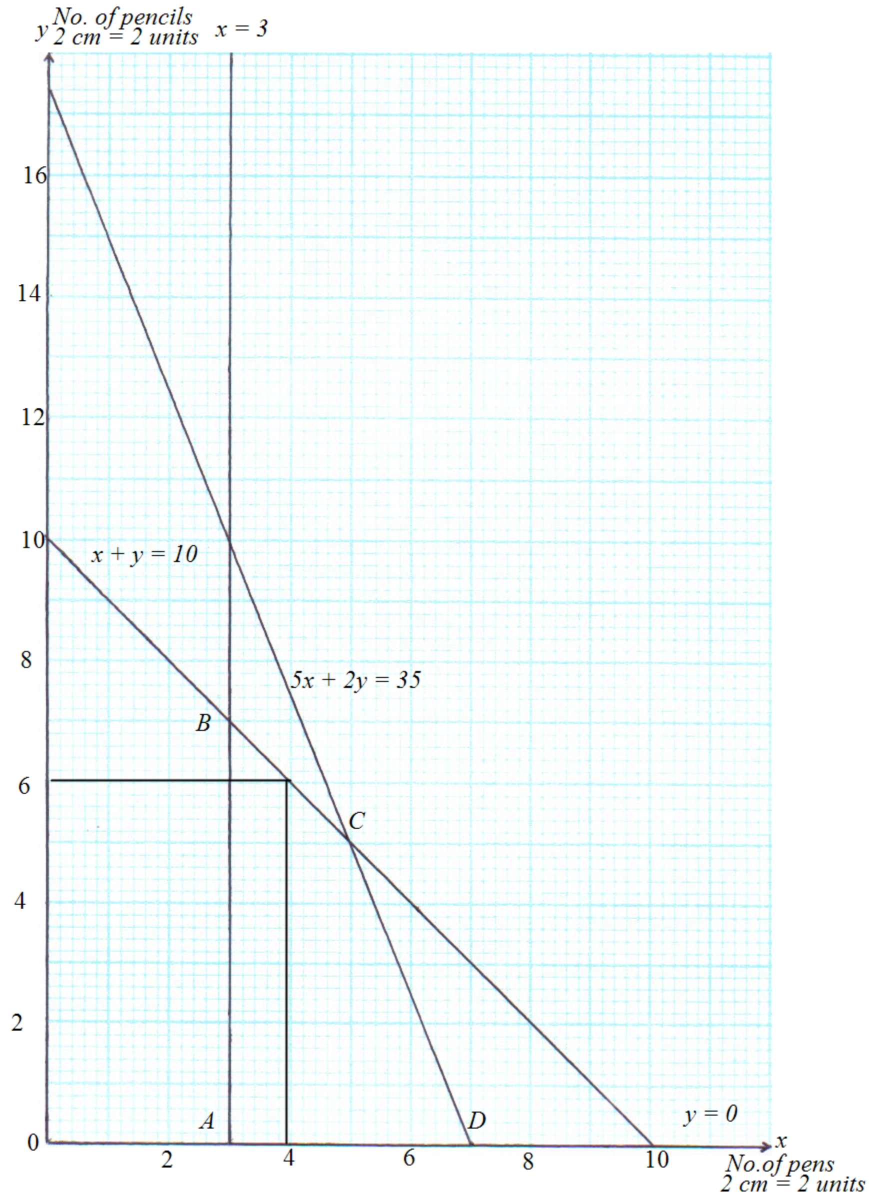
$$P = 5\left(1\frac{1}{2}\right) + 5$$
$$= \$12\frac{1}{2}$$
$$= \$12.50$$

At D $x = 7$ $y = 0$

$$P = 7\left(1\frac{1}{2}\right)$$
$$= \$10\frac{1}{2}$$
$$= \$10.50$$

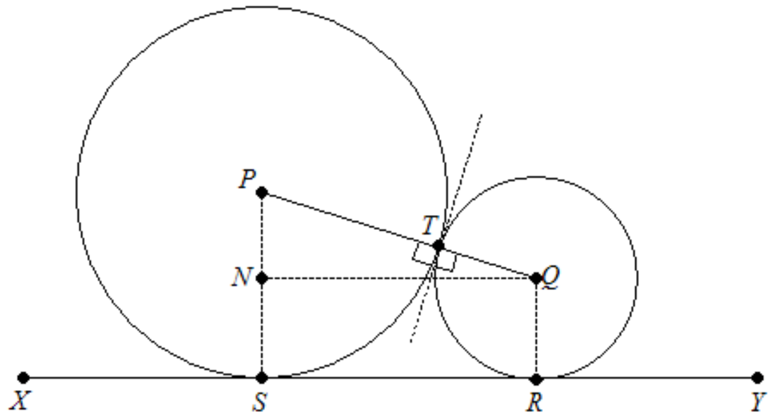
$\therefore$ Maximum profit made is \$12.50 when Pam buys 5 pens and 5 pencils.

(iii) Required To Find: The maximum number of pencils Pam can buy if she buys 4 pens.
Solution:

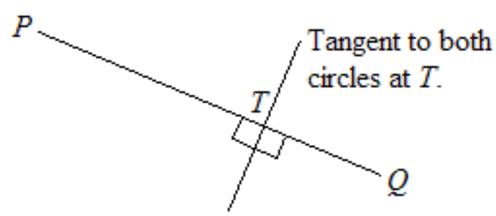


When $x=4$ the maximum value of $y \in Z^{+}$ is 6. Therefore, when 4 pens are bought, the maximum number of pencils that can be bought that satisfies all conditions is 6.

11. a. **Data:** Diagram showing 2 circles of radii 5 cm and 2 cm touching at T, $XSRY$ is a straight line touching the circles at S and R.



(i) (a) **Required To State:** Why PTQ is a straight line.
Solution:
The tangent to both circles at T is a common tangent.



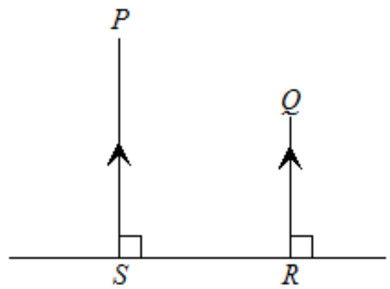
The tangent makes an angle of 90° with the radius PT and 90° with the radius TQ.
(Angle made by a tangent to a circle and the radius, at the point of contact = 90°).
$\therefore P\hat{T}Q = 180°$ (as illustrated) and PTQ is a straight line.

(b) **Required To State:** The length of PQ.
Solution:
Length of PQ = Length of PT + Length of TQ

$$= 5 + 2$$
$$= 7 \text{ cm}$$

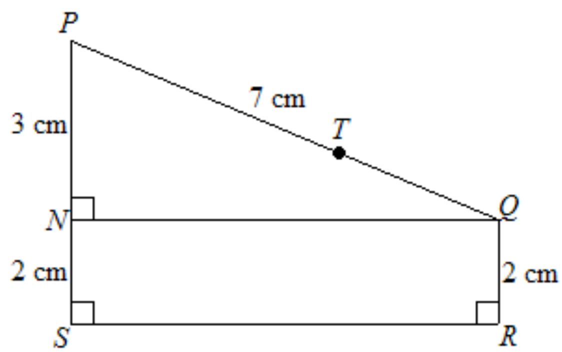
(c) (i) **Required To State:** Why PS is parallel to QR.
Solution:
$P\hat{S}R = Q\hat{R}S = 90°$

(Angle made by a tangent to a circle and the radius, at the point of contact = 90°).



There are corresponding angles, when *PS* is parallel to *QR* and *SR* is a transversal.

(ii) **Data**: N is a point such that QN is perpendicular to PS.



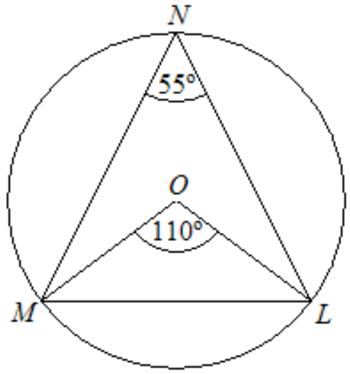
(a) **Required To Calculate:** The length *PN*.
Calculation:
QRSN is a rectangle and hence *NS* = 2 cm, *PS* = 5 cm

$$\therefore PN = 5 - 2$$
$$= 3 \text{ cm}$$

(b) **Required To Calculate:** The length *SR*.
Calculation:

$$NQ = \sqrt{(7)^2 - (3)^2}$$
$$= \sqrt{40} \text{ cm}$$
$$SR = NQ$$
$$SR = \sqrt{40} \text{ cm exactly}$$
$$= 6.32\underline{4} \text{ cm}$$
$$= 6.32 \text{ cm to 2 decimal places}$$

b. **Data:** Circle, centre O and $M\hat{O}L = 110°$.



(i) **Required To Calculate:** $M\hat{N}L$
Calculation:

$$M\hat{N}L = \frac{1}{2}(110°)$$
$$= 55°$$

(Angle subtended by a chord at the centre of a circle is twice the angle it subtends at the circumference, standing on the same arc).

(ii) **Required To Calculate:** $L\hat{M}O$
Calculation:

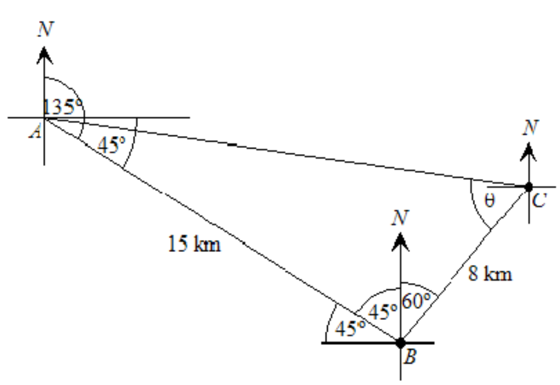
$OM = OL$ (radii)

$$L\hat{M}O = \frac{180° - 110°}{2}$$
$$= 35°$$

(Base angles of an isosceles triangle are equal and sum of the angles in a triangle = 180°).

12. **Data**: The distances and directions of a boat traveling from A to B and then to C.

a. **Required To Draw:** Diagram of the information given, showing the north direction, bearings 135° and 060° and distances 8 km and 15 km.
Solution:



b. (i) **Required To Calculate:** The distance AC.
Calculation:

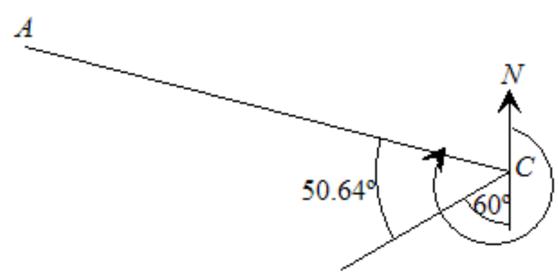
$$A\hat{B}C = 45° + 60°$$
$$= 105°$$
$$AC^2 = (15)^2 + (8)^2 - 2(15)(8)\cos 105° \text{ (cosine law)}$$
$$= 18.73\underline{8} \text{ km}$$
$$= 18.74 \text{ km to 2 decimal places}$$

(ii) **Required To Calculate:** $B\hat{C}A$
Calculation:

Let $B\hat{C}A = \theta$

$$\frac{15}{\sin\theta} = \frac{18.738}{\sin 105°} \text{ (sin law)}$$
$$\therefore \sin\theta = \frac{15\sin 105°}{18.738}$$
$$= 0.7732$$
$$\therefore \theta = \sin^{-1}(0.7732)$$
$$\theta = 50.6\underline{4}°$$
$$= 50.6° \text{ to the nearest } 0.1°$$

(iii) **Required To Calculate:** The bearing from A from C.
Calculation:

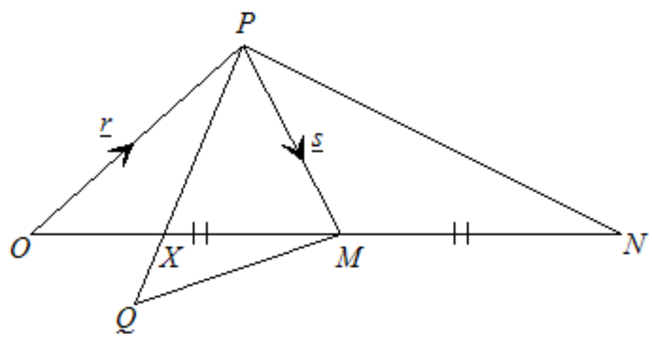


The bearing of A from C $= 180° + 60° + 50.64°$
$= 290.64°$
$= 290.6°$ to the nearest $0.1°$

13. **Data:** Vector diagram with $\overrightarrow{OP} = \underset{\sim}{r}$, $\overrightarrow{PM} = \underset{\sim}{s}$ and OMN a straight line with midpoint M.

$\overrightarrow{OX} = \frac{1}{3}\overrightarrow{OM}$ and $\overrightarrow{PX} = 4\overrightarrow{XQ}$

a. **Required To Sketch:** Diagram illustrating the information given.
Solution:



b. (i) **Required To Express:** $\overrightarrow{OM}$ in terms of $\underset{\sim}{r}$ and $\underset{\sim}{s}$.
Solution:

$$\overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{PM}$$
$$= \underset{\sim}{r} + \underset{\sim}{s}$$

(ii) **Required To Express:** $\overrightarrow{PX}$ in terms of $\underset{\sim}{r}$ and $\underset{\sim}{s}$.
Solution:

$$\overrightarrow{OX} = \frac{1}{3}\overrightarrow{OM}$$
$$= \frac{1}{3}(\underset{\sim}{r} + \underset{\sim}{s})$$
$$\overrightarrow{PX} = \overrightarrow{PO} + \overrightarrow{OX}$$
$$= -(\underset{\sim}{r}) + \frac{1}{3}(\underset{\sim}{r} + \underset{\sim}{s})$$
$$= -\frac{2}{3}\underset{\sim}{r} + \frac{1}{3}\underset{\sim}{s}$$

(iii) **Required To Express:** $\overrightarrow{QM}$ in terms of $\underset{\sim}{r}$ and $\underset{\sim}{s}$.
Solution:

$$\overrightarrow{PX} = 4\overrightarrow{XQ}$$
$$\overrightarrow{PQ} = \frac{5}{4}\overrightarrow{PX}$$
$$= \frac{5}{4}\left(-\frac{2}{3}\underset{\sim}{r} + \frac{1}{3}\underset{\sim}{s}\right)$$

$$\overrightarrow{PQ} = -\frac{5}{6}\underline{r} + \frac{5}{12}\underline{s}$$

$$\overrightarrow{QM} = \overrightarrow{QP} + \overrightarrow{PM}$$

$$= -\left(-\frac{5}{6}\underline{r} + \frac{5}{12}\underline{s}\right) + \underline{s}$$

$$= \frac{5}{6}\underline{r} + \frac{7}{12}\underline{s}$$

c. **Required To Prove:** $\overrightarrow{PN} = 2\overrightarrow{PM} + \overrightarrow{OP}$

Proof:

$$2\overrightarrow{PM} = 2(\underline{s})$$

$$\overrightarrow{OP} = \underline{r}$$

$$2\overrightarrow{PM} + \overrightarrow{OP} = 2\underline{s} + \underline{r}$$

$$= \underline{r} + 2\underline{s}$$

Hence, $\overrightarrow{PN} = 2\overrightarrow{PM} + \overrightarrow{OP} \qquad (= \underline{r} + 2\underline{s})$

Q.E.D

14. a. **Data:** $D = \begin{pmatrix} 1 & 9p \\ p & 4 \end{pmatrix}$

Required To Calculate: p

Calculation:

If $D = \begin{pmatrix} 1 & 9p \\ p & 4 \end{pmatrix}$ is singular then det D = 0.

$$\therefore (1 \times 4) - (9p \times p) = 0$$

$$4 = 9p^2$$

$$p^2 = \frac{4}{9}$$

$$p = \sqrt{\frac{4}{9}}$$

$$= \pm\frac{2}{3}$$

Hence, $p = \pm\frac{2}{3}$.

b. **Data:** $2x+5y=6$ and $3x+4y=8$

(i) **Required To Express:** The above equations in the form $AX=B$.

Solution:

$2x+5y=6$

$3x+4y=8$

Hence, $\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 6 \\ 8 \end{pmatrix}$ …matrix equation

is of the form $AX=B$ where

$A=\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$

$X=\begin{pmatrix} x \\ y \end{pmatrix}$ and

$B=\begin{pmatrix} 6 \\ 8 \end{pmatrix}$ are matrices.

(ii) (a) **Required To Calculate:** Determinant of A.

Calculation:

$$\begin{aligned}\text{Det A} &= (2\times 4)-(5\times 3)\\ &= 8-15\\ &= -7\end{aligned}$$

(b) **Required To Prove:** $A^{-1}=\begin{pmatrix} -\frac{4}{7} & \frac{5}{7} \\ \frac{3}{7} & -\frac{2}{7} \end{pmatrix}$.

Proof:

$$\begin{aligned}A^{-1} &= -\frac{1}{7}\begin{pmatrix} 4 & -(5) \\ -(3) & 2 \end{pmatrix}\\ &= \begin{pmatrix} -\frac{4}{7} & \frac{5}{7} \\ \frac{3}{7} & -\frac{2}{7} \end{pmatrix}\end{aligned}$$

Q.E.D.

(c) **Required To Calculate:** x and y

Calculation:

$$AX = B$$

$\times A^{-1}$

$$A \times A^{-1} \times X = A^{-1} \times B$$

$$I \times X = A^{-1}B$$

$$X = A^{-1}B$$

and

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{4}{7} & \frac{5}{7} \\ \frac{3}{7} & -\frac{2}{7} \end{pmatrix} \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} \left(-\frac{4}{7} \times 6\right) + \left(\frac{5}{7} \times 8\right) \\ \left(\frac{3}{7} \times 6\right) + \left(-\frac{2}{7} \times 8\right) \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{24}{7} + \frac{40}{7} \\ \frac{18}{7} - \frac{16}{7} \end{pmatrix}$$

$$= \begin{pmatrix} 2\frac{2}{7} \\ \frac{2}{7} \end{pmatrix}$$

Equating corresponding $x = 2\frac{2}{7}$ and $y = \frac{2}{7}$.