JUNE 2005 CXC MATHEMATICS GENERAL PROFICIENCY (PAPER 2)

Section I

1. a. **Required To Calculate:** $4\frac{1}{5}-\left(1\frac{1}{9}\times 3\right)$

Calculation:

$$4\frac{1}{5}-\left(1\frac{1}{9}\times 3\right)$$
$$=4\frac{1}{5}-\left(\frac{10}{9}\times 3\right)$$
$$=4\frac{1}{5}-\frac{10}{3}$$
$$=4\frac{1}{5}-3\frac{1}{3}$$
$$=\frac{21}{5}-\frac{10}{3}$$
$$=\frac{3(21)-5(10)}{15}$$
$$=\frac{63-50}{15}$$
$$=\frac{13}{15}$$
$$=\frac{13}{15}\text{ (in exact form)}$$

b. **Data:** Table showing Amanda's shopping bill

(i) **Required To Calculate:** The values of A, B, C and D
Calculation:
3 T-shirts at \$12.50 each cost a total of
$3\times \$12.50=\$37.50$
$\therefore A=\$37.50$

2 CD's cost a total of \$33.90
$\therefore$ The unit price is $\frac{\$33.90}{2}$
$=\$16.95$
$\therefore B=\$16.95$

C posters at \$6.20 each cost \$31.00

$\therefore C = \dfrac{\$31.00}{\$6.00}$

$\therefore C = 5$

The total bill is \$108.28

$\therefore 15\%\ \text{VAT} = \dfrac{15}{100} \times \$108.28$

$= \$16.24\underline{2}$

$= \$16.24$ to the nearest *cent*

$\therefore D = \$16.24$

(ii) **Required To Determine:** Whether Amanda made a profit or a loss
Solution:
Price paid for 6 stickers at \$0.75 each and 6 stickers at \$0.40 each

$= (6 \times 0.75) + (6 \times 0.40)$

$= \$4.50 + \$2.40$

$= \$6.90$

The cost of 12 stickers to Amanda = \$5.88
Since the selling price > Cost price, then Amanda acquired a profit of

$(\$6.90 - \$5.88)$

$= \$1.02$

2. a. **Required To Factorise:** (i) $5a^2b + ab^2$, (ii) $9k^2 - 1$, (iii) $2y^2 - 5y + 2$
Factorising:

(i) $5a^2b + ab^2$

$= 5.a.a.b + a.b.b$

$= ab(5a + b)$

(ii) $9k^2 - 1$

$= (3k)^2 - (1)^2$

This is the difference of two squares

$(3k - 1)(3k + 1)$

(iii) $2y^2 - 5y + 2$

$(2y - 1)(y - 2)$

b. **Required To Simplify:** $(2x+5)(3x-4)$
Solution:
Simplifying $(2x+5)(3x-4)$
$=6x^2+15x-8x-20$
$=6x^2+7x-20$

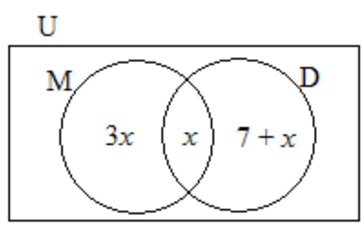
c. **Data:** Card game played among 3 people.
Solution:
Score by Adam $=x$ points

Imran's score is 3 less than Adam's score $=(x-3)$ (data)

(i) **Required To Find:** an expression in terms of x for the number of points scored by Shakeel.
Solution:
Shakeel's score is 2 times Imran's score $=2(x-3)$ points

(ii) **Required To Find:** an equation which may be used to find the value of x.
Solution:
Total score = 39 points
$\therefore x+(x-3)+2(x-3)=39$
$x+x-3+2x-6=39$
$4x-9=39$
$4x=48$
and $x=12$

3. a. **Data:** Venn diagram illustrating the students in a class who study Music and /or Dance.
Solution:



(i) **Required To Calculate:** the number of students who take both Music and Drama.
Calculation:
$n(M)=24$ (data)

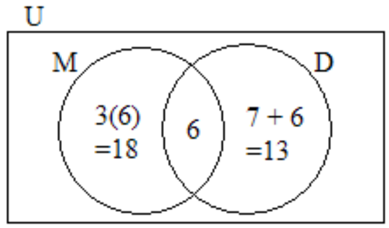
$\therefore 3x + x = 24$

$4x = 24$

and $x = 6$

And $n(M \cap D)$, that is number of students who take both Music and Dance = 6

(ii) **Required To Calculate:** the number of students who take Drama only.
Calculation:
Hence

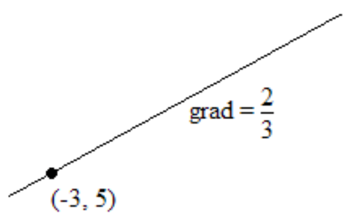


$n(D \text{ only}) = 13$

That is, the number of students who take Dance only = 13

b. **Data:** Line with gradient $\frac{2}{3}$ passes through *P* (-3, 5)

(i) **Required To Find:** the equation of the line through *P* (-3, 5) and with gradient $\frac{2}{3}$.
Solution:



Equation of line is

$$\frac{y-5}{x-(-3)} = \frac{2}{3}$$

$$3y - 15 = 2x + 6$$

$$3y = 2x + 21$$

$$y = \frac{2}{3}x + 7$$

is of the form $y = mx + c$, where $m = \frac{2}{3}$ and $c = 7$.

(ii) **Required To Prove:** the above line is parallel to the line $2x-3y=0$

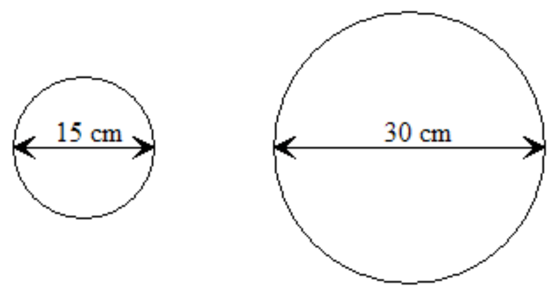
Solution:

$$2x-3y=0$$
$$3y=2x$$
$$y=\frac{2}{3}x$$

is of the form $y=mx+c$, where $m=\frac{2}{3}$ is the gradient.

Hence $y=\frac{2}{3}x+7$ and $2x-3y=0$ are parallel since they both have the same gradient $\left(=\frac{2}{3}\right)$ and parallel lines have the same gradient.

4. **Data:** Diagrams of



Small pizza Medium pizza

a. **Required To Determine:** Whether a medium pizza is twice as large as a small pizza.

Solution:

The pizzas are 3-dimensional, hence a comparison of sizes must be made by comparing their volumes. Both have the same height (thickness).

Volume of small pizza

$$V_s=\pi r^2h$$
$$=\pi(7.5)^2h$$
$$=56.25\pi h\text{ cm}^3$$

Volume of medium pizza

$$V_m=\pi r^2h$$
$$=\pi(15)^2h$$
$$=225\pi h\text{ cm}^3$$
$$=4(56.25\pi h)$$

So we see that the medium pizza has 4 times the volume of a small pizza. So that statement – A medium pizza is twice as large as a small pizza is **INCORRECT.**

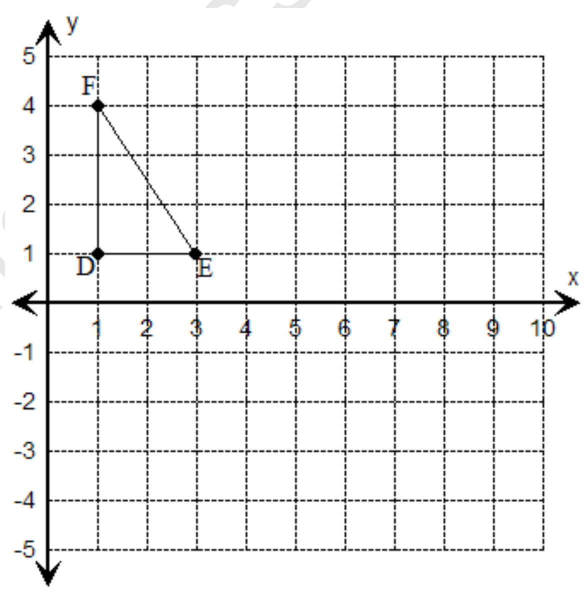
b. **Data:** The prices for each slice of a medium pizza and for one small pizza.
Required To Find: Whether it is better to buy 1 medium pizza or 4 small pizzas.
Solution:

$$\text{Cost of } \frac{1}{3} \text{ medium pizza} = \$15.95$$

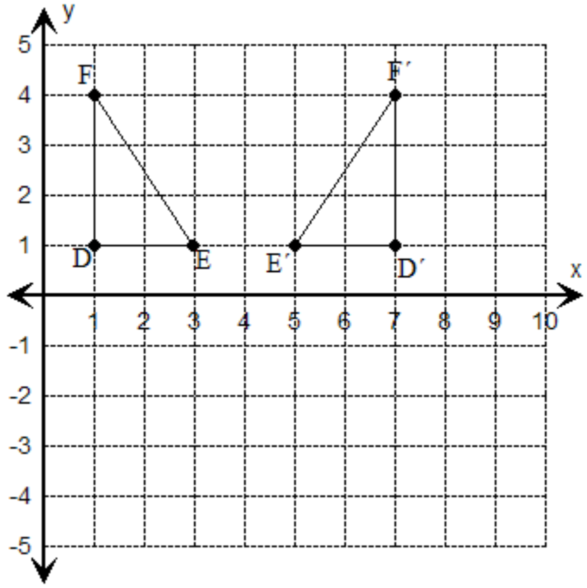
$$\therefore \text{Cost of an entire medium pizza} = \$15.95 \times 3$$
$$= \$47.85$$

Cost of 1 small pizza = \$12.95
Since 4 small pizzas $\equiv$ 1 medium pizza
Then the equivalent cost of 4 small pizzas
$= \$12.95 \times 4$
$= \$51.80$
4 small pizzas is equivalent in volume to 1 medium pizza which costs \$47.95
$\therefore$ The 'better buy' (which supposedly means **more pizza** at a lesser price) is obtained by buying a medium pizza.

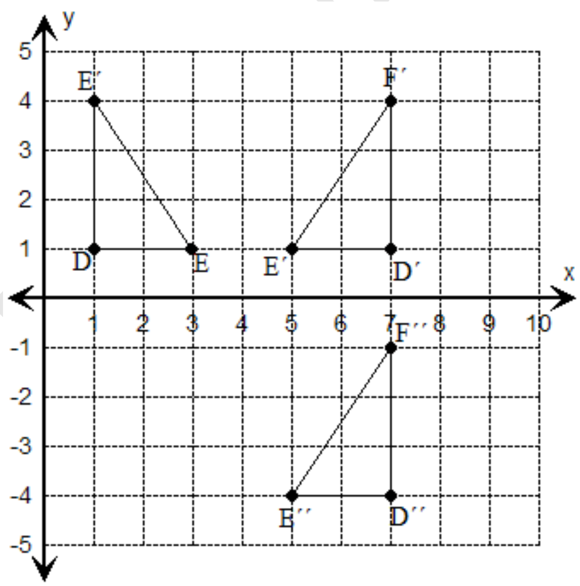
5. a. **Data:** The coordinates of the vertices of a triangle, *D*, *E* and *F*.
Required To Draw: ΔDEF .
Solution:



b. (i) **Required to draw:** $\Delta D'E'F'$
Solution:
$D' = (7,1) \qquad E' = (5,1) \qquad F' = (7,4)$



(ii) **Required To Draw:** $\Delta D''E''F''$
Solution:
$D''=(7,-4)\quad E''=(5,-4)\quad F''=(7,-1)$

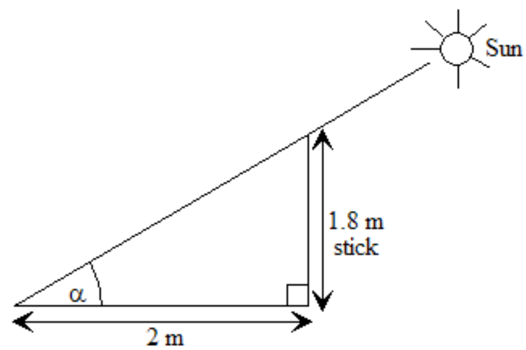


(iii) **Required to identify:** the type of transformation that maps ΔDEF onto $\Delta D''E''F''$
Solution:

$$\Delta DEF \xrightarrow{\text{Reflection in } x=4} \Delta D'E'F' \xrightarrow{\text{Translation}=\begin{pmatrix}0\\-5\end{pmatrix}} \Delta D''E''F''$$

Hence $\Delta DEF \xrightarrow{\text{Glide reflection}} \Delta D''E''F''$

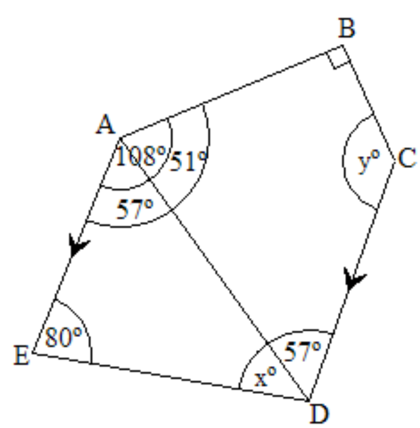
c. **Data:** Stick 1.8 m casts a shadow 2 m long.



Required To Calculate: Angle of elevation of the sun
Calculation:
Let the angle of elevation be α

$$\tan\alpha = \frac{1.8}{2}$$
$$\alpha = \tan^{-1}(0.9)$$
$$\alpha = 41.9^\circ$$
$$\alpha = 42^\circ \text{ (to the nearest degree)}$$

6. a. **Data:** Diagram of a pentagon ABCDE
Required To Calculate: x°, y°
Calculation:



(i) $E\hat{A}D = 57^\circ$ (alternate angles)
$x^\circ = 180^\circ - (80^\circ + 57^\circ)$
$= 43^\circ$

(Sum of angles in a triangle = 180°)

(ii) $B\hat{A}D = 108° - 57°$
$= 51°$
$\therefore y = 360° - (51° + 80° + 57°)$
$= 162°$
(Sum of angles in a quadrilateral = 360°)

b. **Data:** $f(x) = \frac{1}{2}x + 5$ $\quad g(x) = x^2$

(i) **Required To Evaluate:** $g(3) + g(-3)$
Solution:
$g(3) + g(-3)$
$= (3)^2 + (-3)^2$
$= 9 + 9$
$= 18$

(ii) **Required To Evaluate:** $f^{-1}(6)$
Solution:
Let $y = \frac{1}{2}x + 5$
$y - 5 = \frac{1}{2}x$
$2y - 10 = x$
Replace y by x
$f^{-1}(x) = 2x - 10$
$\therefore f^{-1}(6) = 2(6) - 10$
$= 2$

(iii) **Required To Evaluate:** $fg(2)$
Solution:
$g(2) = (2)^2$
$= 4$
$fg(2) = f(4)$
$= \frac{1}{2}(4) + 5$
$= 2 + 5$
$= 7$

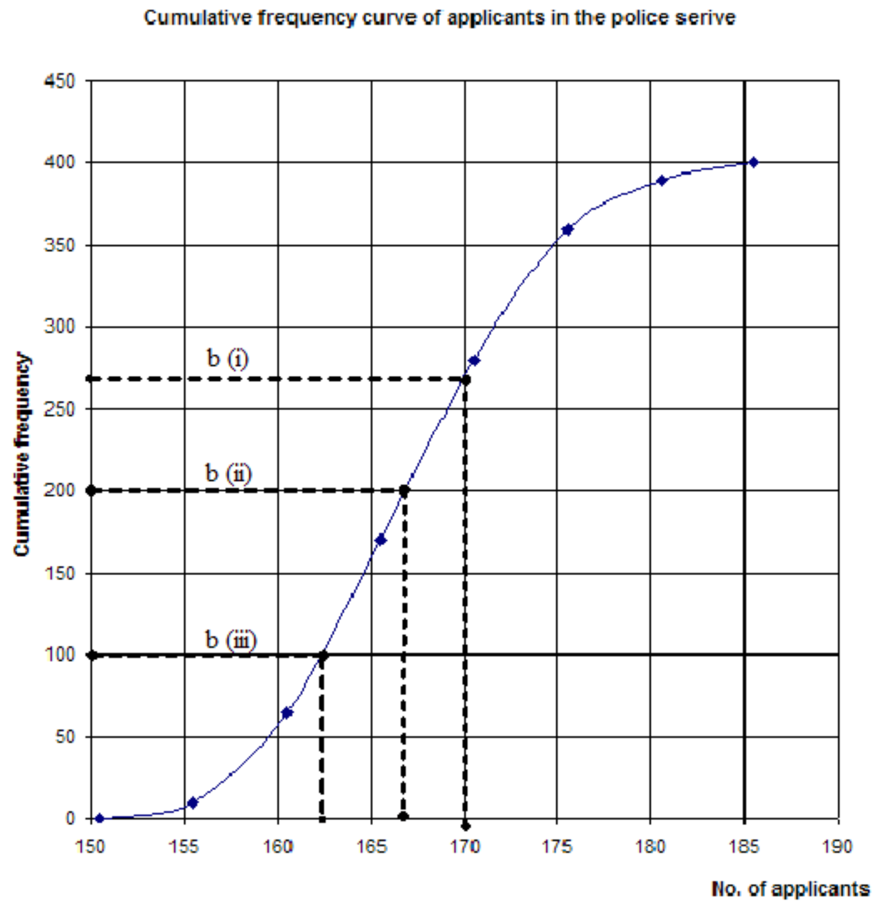
7. **Data:** Table showing the height of 400 applicants for the police service.

a. **Required To Draw:** the cumulative frequency curve of heights given in the table.
Solution:
The data shows a Continuous variable and we create the table as:

Height in cm, x	L.C.B. U.C.B	No. of applicants	Cumulative frequency	Points to be plotted (U.C.B, CF)
151 – 155	$150.5 \le x < 155.5$	10	10	(150.5, 0) (155.5, 10)
156 – 160	$155.5 \le x < 160.5$	55	65	(160.5, 65)
161 – 165	$160.5 \le x < 165.5$	105	170	(165.5, 170)
166 – 170	$165.5 \le x < 170.5$	110	280	(170.5, 280)
171 – 175	$170.5 \le x < 175.5$	80	360	(175.5, 360)
176 – 180	$175.5 \le x < 180.5$	30	390	(180.5, 390)
181 – 185	$180.5 \le x < 185.5$	10	400	(185.5, 400)

$$\sum f = 400$$

The point (150.5, 0) is obtained by extrapolation, so as to start the curve on the horizontal axis.



b. (i) **Required To Estimate:** the number of applicants whose heights are less than 170 cm.
Solution:
From the graph, $\approx$ 265 applicants are less than 170 cm (read off).

(ii) **Required to estimate:** the median height of applicants.
Solution:
The median height of applicants $\approx$ 167 cm (read off).

(iii) **Required to estimate:** the height that 25% of the applicants are less than
Solution:

$$25\% \text{ of the applicants } = \frac{25}{100} \times 400$$
$$= 100$$

100 applicants are less than 162 cm (read off).

(iv) **Required To Estimate:** the probability that a randomly selected applicant has a height no more than 162 cm.

Solution:

P(applicant's height is no more than 162 cm)

$$= \frac{\text{No. of applicants} \leq 162\,\text{cm}}{\text{No. of applicants}}$$

$$= \frac{100}{400}$$

$$= \frac{1}{4}$$

8. a. **Data:** Table showing a number pattern

Required To Complete: the table given.

Solution:

2^3	$(0\times 3^2)+(3\times 2)+2$	8
3^3	$(1\times 4^2)+(3\times 3)+2$	27
4^3	$(2\times 5^2)+(3\times 4)+2$	64
5^3	$(3\times 6^2)+(3\times 5)+2$ 2 less than the number that is being cubed. 1 more than the number that is being cubed always 3 the number that is being cubed always 2	125 the result
(i) 6^3	$(6-2)\times(6+1)^7+(3\times 6)+2$ $=(4\times 7^2)+(3\times 6)+2$	216
⋮	⋮	⋮
(ii) 10^3	$(10-2)\times(10+1)^2+(3\times 10)+2$ $=(8\times 11^2)+(3\times 10)+2$	1000
⋮	⋮	⋮
(iii) n^3	$(n-2)\times(n+1)^2+(3\times n)+2$	n^3

b. **Required to prove:** $(a-b)^2(a+b)+ab(a+b)=a^3+b^3$

Proof: L.H.S.

$(a-b)^2(a+b)+ab(a+b)$

$=(a^2-2ab+b^2)(a+b)+ab(a+b)$

$=a^3-2a^2b+ab^2+a^2b-2ab^2+b^3+a^2b+ab^2$

$=a^3+b^3$

$=$ R.H.S.

Q.E.D.

Section II

9. a. **Required to express:** $5x^2+2x-7$ in the form $a(x+b)^2+c$, $a,b,c\in\Re$

Solution:

$a(x+b)^2+c$

$=a(x^2+2bx+b^2)+c$

$=ax^2+2abx+ab^2+c$

Equating the coefficient of x^2

$a=5 \quad \in\Re$

Equating the coefficient of x

$2(5)b=2$

$b=\frac{1}{5} \quad \in\Re$

Equating constants

$5\left(\frac{1}{5}\right)^2+c=-7$

$\frac{1}{5}+c=-7$

$c=-7\frac{1}{5} \quad \in\Re$

$\therefore 5x^2+2x-7\equiv 5\left(x+\frac{1}{5}\right)^2-7\frac{1}{5}$

OR

$5x^2+2x-7$

$=5\left(x^2+\frac{2}{5}x\right)-7$

Half the coefficient of x is $\frac{1}{2}\left(\frac{2}{5}\right)=\frac{1}{5}$

$=5\left(x^2+\frac{2}{5}x+\frac{1}{25}\right)$

$=5x^2+2x+\frac{1}{5}$

$-7\frac{1}{5}$

-7

$=5\left(x+\frac{1}{5}\right)^2-7\frac{1}{5}$

is of the form $a(x+b)^2+c$, where

$a=5\in\Re$

$b=\frac{1}{5}\in\Re$

$c=-7\frac{1}{5}\in\Re$

b. (i) **Required To Determine:** the minimum value of $y=5x^2+2x-7$

Solution:

$y=5x^2+2x-7$

and

$y=5\left(x+\frac{1}{5}\right)^2-7\frac{1}{5}$

$5\left(x+\frac{1}{5}^2\right)\geq 0 \quad \forall x$

$\therefore y_{\min}=0-7\frac{1}{5}$

$=-7\frac{1}{5}$

(ii) **Required To Determine:** the value of x at which the minimum point occurs.

Solution:

When

$$5\left(x+\frac{1}{5}\right)^2=0$$

$$\left(x+\frac{1}{5}\right)^2=0$$

$$\left(x+\frac{1}{5}\right)=0$$

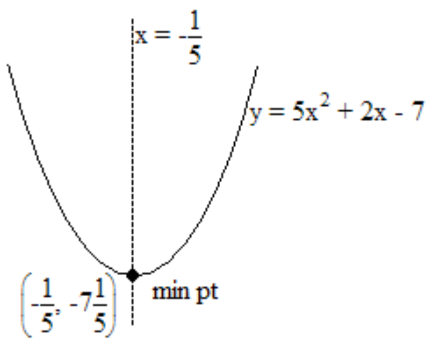
$$x=-\frac{1}{5}$$

OR

$y=5x^2+2x-7$ has an axis of symmetry at

$$x=\frac{-(2)}{2(5)}$$

$$=-\frac{1}{5}$$



At minimum point $x=-\frac{1}{5}$ and $y=5\left(-\frac{1}{5}\right)^2+2\left(-\frac{1}{5}\right)-7$

$$=-7\frac{1}{5}$$

$y_{\min}=-7\frac{1}{5}$ at $x=-\frac{1}{5}$

c. **Required To Solve:** $5x^2+2x-7=0$

Solution:

$$5x^2+2x-7=0$$
$$(5x+7)(x-1)=0$$
$$\therefore x=1 \text{ or } -\frac{7}{5}$$

OR

$$x=\frac{-(2)\pm\sqrt{(2)^2-4(5)(-7)}}{2(5)}$$
$$=\frac{-2\pm\sqrt{4+140}}{10}$$
$$=\frac{-2\pm\sqrt{144}}{10}$$
$$=\frac{-2\pm 12}{10}$$
$$=-\frac{14}{10} \text{ or } \frac{10}{10}$$
$$=1 \text{ or } -1\frac{2}{5}$$

OR

$$5x^2+2x-7=0$$
$$5\left(x+\frac{1}{5}\right)^2-7\frac{1}{5}=0$$
$$5\left(x+\frac{1}{5}\right)^2=\frac{36}{5}$$
$$\left(x+\frac{1}{5}\right)^2=\frac{36}{25}$$

Find the square root

$$\left(x+\frac{1}{5}\right)=\pm\frac{6}{5}$$
$$x=-\frac{1}{5}\pm\frac{6}{5}$$
$$=\frac{-1\pm 6}{5}$$
$$=-\frac{7}{5} \text{ or } 1$$

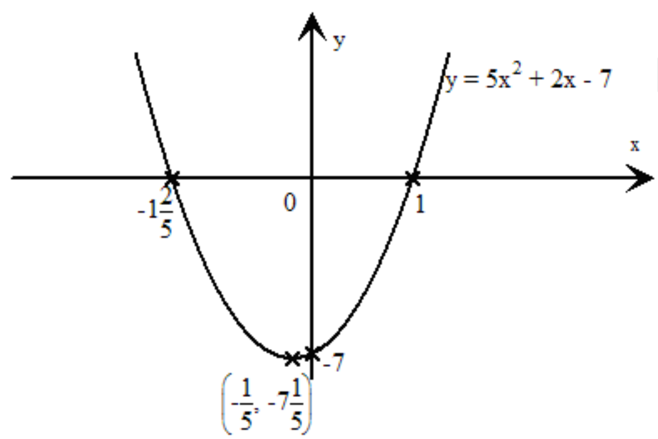
c. **Required To Sketch:** the graph of $y = 5x^2 + 2x - 7$, showing the coordinates of the minimum point, the value of the y – intercept and the points where the graph cuts the x – axis.

Solution:

When $x = 0 \quad y = 5(0)^2 + 2(0) - 7 = -7$

$\therefore$ Curve cuts the y – axis at (0, -7) and the x – axis at 1 and $-\frac{7}{5}$.

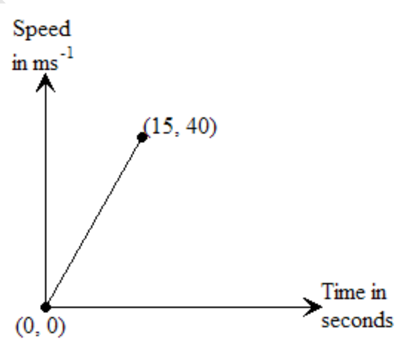
Minimum point = $\left(-\frac{1}{5}, -7\frac{1}{5}\right)$



10. a. **Data:** Speed – time graph for the movement of a cyclist.

(i) **Required To Calculate:** The acceleration of the cyclist during the first 15 seconds.

Calculation:

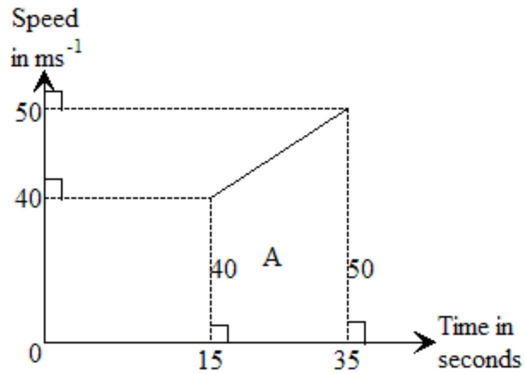


Since the branch for the first 15 seconds of the journey is a straight line, then the acceleration is constant.

Gradient $= \frac{40 - 0}{15 - 0} \quad = 2\frac{2}{3}$ $\qquad \therefore$ Acceleration $= 2\frac{2}{3}\text{ ms}^{-2}$

(ii) **Required To Calculate:** The distance travelled by the cyclist between $t = 15$ and $t = 35$.

Calculation:



The distance covered between $t = 15$ and $t = 35$ is the area of the region, A, shown in the diagram which describes a trapezium.

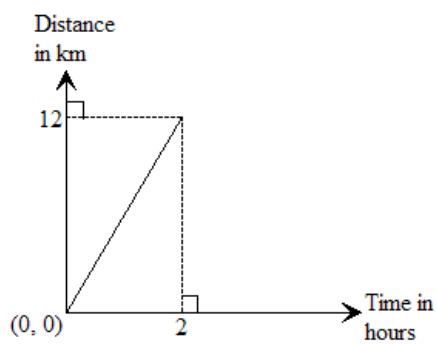
$$= \frac{1}{2}(40 + 50) \times (35 - 15)$$

$$= 900 \text{ m}$$

b. **Data:** Diagram showing the distance – time journey of an athlete

(i) **Required To Calculate:** The average speed during the first 2 hours.

Calculation:



The average speed during the first 2 hours

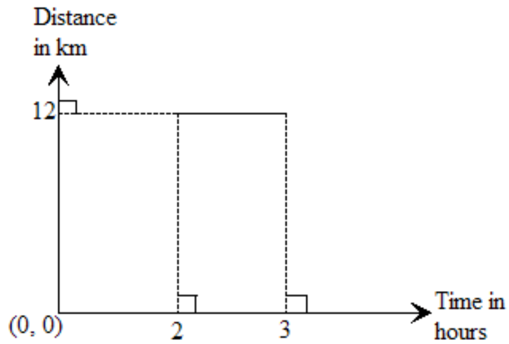
$$= \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{12 \text{ km}}{2 \text{ h}}$$

$$= 6 \text{ kmh}^{-1}$$

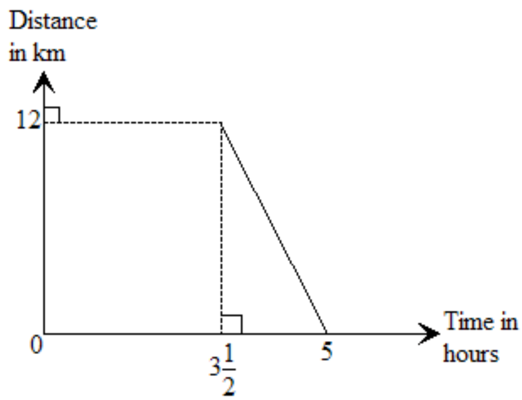
(ii) **Required To Determine:** What the athlete did between 2 and 3 hours after the start of the journey.

Solution:



At $t = 2$,distance = 12 km and at $t = 3$, distance = 12 km.
This is a indicated by a horizontal branch in the graph. Hence, between 2 and 3 hours after the start, the cyclist did NOT travel **OR** the cyclist stopped cycling for that 1 hour interval.

(iii) **Required To Calculate:** the average speed on the return journey.
Solution:



The return journey took $5 - 3\frac{1}{2}$

$$= 1\frac{1}{2} \text{ hours.}$$

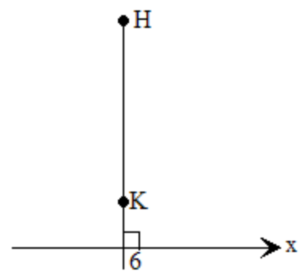
$$\therefore \text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{12 \text{ km}}{1\frac{1}{2} \text{ h}}$$

$$= 8 \text{ kmh}^{-1}$$

(c) **Data:** Diagram of a triangle bounded by lines *GH*, *GK* and *HK*.

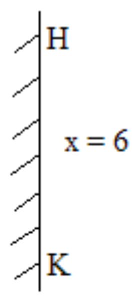
(i) **Required To Find:** the equation of the line *HK*
Solution:



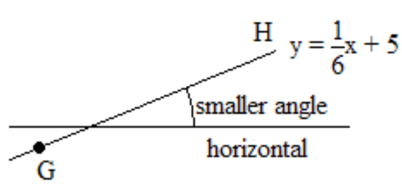
HK is a vertical line that cuts the $x-$axis at 6. Therefore, the equation of HK is $x=6$.

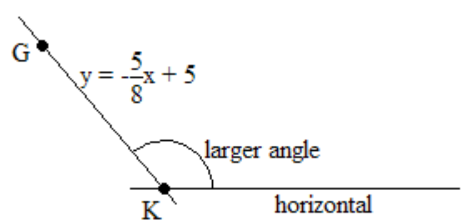
(ii) **Required to find:** the set of 3 inequalities which define the shaded region in the diagram.
Solution:

Region shaded is on the left of $x=6$. Hence, $x \le 6$ (and including line).



The region shaded is on the side with the smaller angle. Hence,
$y \le \frac{1}{6}x+5$ (including line).





The side shaded is that with the larger angle. Therefore, region is $y \ge -\frac{5}{8}x+5$ (including line). Hence the three inequalities that define the shaded region are:

$x \le 6$

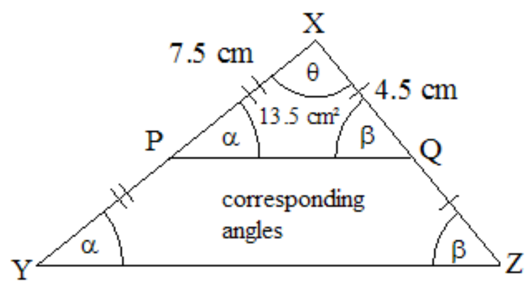
$y \le \frac{1}{6}x+5$

$y \ge -\frac{5}{8}x+5$

11. a. **Data:** P, Q are midpoints of ΔXYZ with $XP = 7.5$ cm, $XQ = 4.5$ cm and area of $\Delta XPQ = 13.5\ \text{cm}^2$

(i) **Required To Calculate:** the size of $\angle PXQ$

Calculation:



Let $P\hat{X}Q = \theta$

$$\therefore \frac{1}{2}(7.5)(4.5)\sin\theta = 13.5$$

$$\therefore \sin\theta = \frac{13.5\times 2}{7.5\times 4.5}$$
$$= 0.8$$
$$\theta = 53.\underline{1}^\circ$$
$$= 53^\circ \text{ (to the nearest degree)}$$

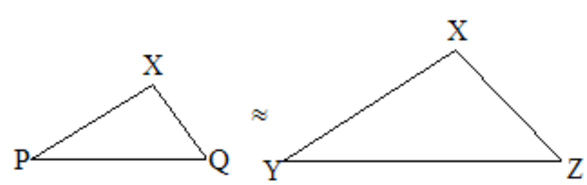
(ii) **Required To Calculate:** Area of ΔYXZ

Calculation:

$$XY = 2(7.5)$$
$$= 15$$
$$XZ = 2(4.5)$$
$$= 9$$
$$\sin\theta = \frac{4}{5}\, or\, 0.8$$

$$\therefore \text{Area of } \Delta YXZ = \frac{1}{2}(15)(9)\times\frac{4}{5}$$
$$= 54 \text{ square units}$$

OR



ΔXPQ and ΔXYZ are equivalent or similar.

$XP : XY = 1 : 2$

$\therefore$ Area of ΔXPQ : Area of $\Delta XYZ = 1^2 : 2^2$

$= 1 : 4$

$\therefore$ Area of $\Delta XYZ = 13.5 \times 4$

$= 54 \text{ cm}^2$

b. **Data:** Diagram of trapezium $SJKM$ with SJ parallel to MK, $SM = SJ = 50\,\text{m}$, $M\hat{J}K = 124°$ and $M\hat{S}T = 136°$

Required To Calculate:

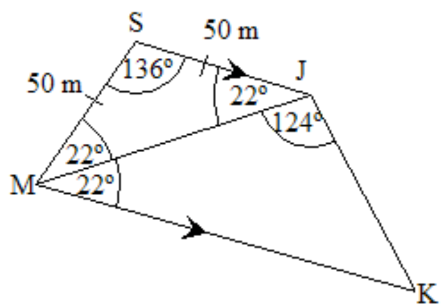
(i)(a) $S\hat{J}M$

(b) $J\hat{K}M$

(ii)(a) MJ

(b) JK

Calculation:



(i) (a) $S\hat{J}M = S\hat{M}J$ (base angles of isosceles triangle)

$$\therefore S\hat{J}M = \frac{180° - 136°}{2}$$

$= 22°$ (sum of angles in $\Delta = 180°$)

(b) $J\hat{M}K = 22°$ (alternate angles)

$\therefore J\hat{K}M = 180° - (124° + 22°)$

$= 34°$ (sum of angles in $\Delta = 180°$)

(ii) (a) $$\frac{MJ}{\sin 136°} = \frac{50}{\sin 22°} \text{ (sine rule)}$$

$$\therefore MJ = \frac{50 \times \sin 136°}{\sin 22°}$$

$= 92.7\underline{1}$ m

$= 92.7$ m to 1 decimal place

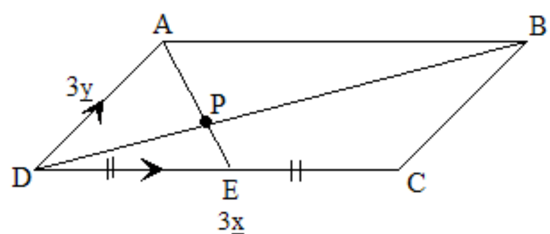
OR

$$MJ^2 = (50)^2 + (50)^2 - 2(50)(50)\cos 136° \quad \text{(Cosine Rule)}$$
$$MJ = 92.7\underline{1}$$
$$= 92.7 \text{ m to 1 decimal place}$$

(b) $$\frac{JK}{\sin 22°} = \frac{92.71}{\sin 34°} \quad \text{(Sine Rule)}$$
$$JK = \frac{92.71 \times \sin 22°}{\sin 34°}$$
$$= 62.1\underline{0} \text{ m}$$
$$= 62.1 \text{ m to 1 decimal place}$$

12. **This question is not done since it involves latitude and longitude (Earth Geometry) which has been removed from the syllabus.**

13. a. **Data:** $ABCD$ is a parallelogram with $\overrightarrow{DC} = 3\underline{x}$, $\overrightarrow{DA} = 3\underline{y}$ and P on DB such that $DP : PB = 1 : 2$.



(i) **Required To Express:** $\overrightarrow{AB}$ in terms of $\underline{x}$ and $\underline{y}$

Solution:

$\overrightarrow{AB} \equiv \overrightarrow{DC}$

(Equal in magnitude and parallel, as expected for opposite sides of a parallelogram).

$\therefore \overrightarrow{AB} = 3\underline{x}$

(ii) **Required To Express:** $\overrightarrow{BD}$ in terms of $\underline{x}$ and $\underline{y}$

Solution:

Similarly as (a) $\overrightarrow{CB} \equiv \overrightarrow{DA} = 3\underline{y}$

$$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$$
$$= -(3\underline{y}) + (-3\underline{x})$$
$$= -3\underline{x} - 3\underline{y}$$

(iii) **Required To Express:** $\overrightarrow{DP}$ in terms of $\underline{x}$ and $\underline{y}$

Solution:

$\overrightarrow{DB} = -\left(-3\underline{x} - 3\underline{y}\right)$

$= 3\underline{x} + 3\underline{y}$

Since $DP : PB = 1:2$, then $DP = \frac{1}{3}DB$ and

$\overrightarrow{DP} = \frac{1}{3}\left(3\underline{x} + 3\underline{y}\right)$

$= \underline{x} + \underline{y}$

b. **Required To Prove:** $\overrightarrow{AP} = \underline{x} - 2\underline{y}$

Proof:

$\overrightarrow{AP} = \overrightarrow{AD} + \overrightarrow{DP}$

$= -\left(3\underline{y}\right) + \left(\underline{x} + \underline{y}\right)$

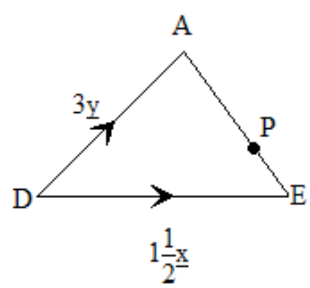
$= \underline{x} - 2\underline{y}$

Q.E.D.

c. **Data:** *E* is the midpoint of *DC*

Required To Prove: *A*, *P* and *E* are collinear.

Solution:

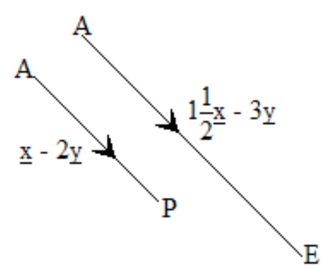


$\overrightarrow{DE} = \frac{1}{2}\left(3\underline{x}\right)$

$\overrightarrow{AE} = \overrightarrow{AD} + \overrightarrow{DE}$

$= -\left(3\underline{y}\right) + 1\frac{1}{2}\underline{x}$

$= 1\frac{1}{2}\underline{x} - 3\underline{y}$



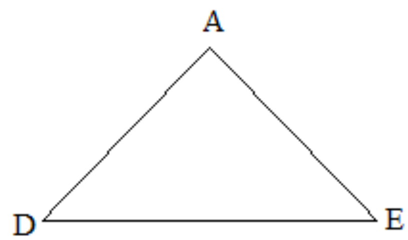
$1\frac{1}{2}\underline{x} - 3\underline{y} = 1\frac{1}{2}\left(\underline{x} - 2\underline{y}\right)$

$= 1\frac{1}{2}\overrightarrow{AP}$

$\overrightarrow{AE}$ is a scalar multiple of $\overrightarrow{AP}$. Therefore, $\overrightarrow{AE}$ is parallel to $\overrightarrow{AP}$. Since A is a common point, P lies on AE and A, P and E are collinear.

d. **Data:** $x = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $y = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Required To Prove: ΔAED is isosceles.
Proof:



$\overrightarrow{DA} = 3\underline{y}$

$= 3\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

$\therefore \left|\overrightarrow{DA}\right| = \sqrt{(3)^2 + (3)^2}$

$= \sqrt{18}$

$\overrightarrow{DE} = 1\frac{1}{2}\underline{x}$

$= 1\frac{1}{2}\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$\left|\overrightarrow{DE}\right| = \sqrt{(3)^2 + (0)^2}$

$= 3$

$\overrightarrow{AE} = 1\frac{1}{2}\underline{x} - 3\underline{y}$

$= 1\frac{1}{2}\begin{pmatrix} 2 \\ 0 \end{pmatrix} - 3\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -3 \end{pmatrix}$

$\left|\overrightarrow{AE}\right| = \sqrt{(0)^2 + (-3)^2}$

$= 3$

In ΔAED only 2 sides, AE and DE, are equal, therefore the triangle is isosceles.

Q.E.D

12. a. **Data:** $M = \begin{pmatrix} 2 & 5 \\ 7 & 15 \end{pmatrix}$

(i) **Required To Prove:** M is a non-singular matrix
Solution:
$$\text{Det } M = (2\times 15) - (5\times 7)$$
$$= 30 - 35$$
$$= -5 \neq 0$$
Hence, $\exists \ M^{-1}$ and so M is non-singular.

(ii) **Required To Find:** M^{-1}
Solution:
$$M^{-1} = -\frac{1}{5}\begin{pmatrix} 15 & -(5) \\ -(7) & 2 \end{pmatrix}$$
$$= \begin{pmatrix} -3 & 1 \\ \frac{7}{5} & -\frac{2}{5} \end{pmatrix}$$

(iii) **Required To Find:** $M \times M^{-1}$
Solution:
$M \times M^{-1} = I$ where I is the 2×2 identity matrix $= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
$$M_{2\times 2} \times M^{-1}{}_{2\times 2} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$
$$\begin{pmatrix} 2 & 5 \\ 7 & 15 \end{pmatrix} \times \begin{pmatrix} -3 & 1 \\ \frac{7}{5} & -\frac{2}{5} \end{pmatrix} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$$
$$e_{11} = (2\times -3) + \left(5 \times \frac{7}{5}\right)$$
$$= -6 + 7$$
$$= 1$$

$$e_{12} = (2\times 1) + \left(5\times -\frac{2}{5}\right)$$
$$= 2 - 2$$
$$= 0$$

$$e_{21} = (7\times -3) + \left(15\times \frac{7}{5}\right)$$
$$= -21 + 21$$
$$= 0$$

$$e_{22} = (7\times 1) + \left(15\times -\frac{2}{5}\right)$$
$$= 7 - 6$$
$$= 1$$

$$\therefore M\times M^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$M\times M^{-1} = I$$

(iv) **Required To Solve:** $\begin{pmatrix} 2 & 5 \\ 7 & 15 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 17 \end{pmatrix}$

Solution:

$$\begin{pmatrix} 2 & 5 \\ 7 & 15 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 17 \end{pmatrix}$$

$\times M^{-1}$

$$\begin{pmatrix} 2 & 5 \\ 7 & 15 \end{pmatrix}\times M^{-1}\begin{pmatrix} x \\ y \end{pmatrix} = M^{-1}\begin{pmatrix} -3 \\ 17 \end{pmatrix}$$

$$I\times\begin{pmatrix} x \\ y \end{pmatrix} = M^{-1}\begin{pmatrix} -3 \\ 17 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ \frac{7}{5} & -\frac{2}{5} \end{pmatrix}\begin{pmatrix} -3 \\ 17 \end{pmatrix}$$
$$= \begin{pmatrix} (-3\times -3) + (1\times 17) \\ \left(\frac{7}{5}\times -3\right) + \left(-\frac{2}{5}\times 17\right) \end{pmatrix}$$
$$= \begin{pmatrix} 26 \\ -11 \end{pmatrix}$$

Equating corresponding entries

$x = 26$ and $y = -11$

b. (i) **Required To Find:** matrix R, which represents a reflection in the y – axis.
Solution:
The matrix, R, which represents a reflection in the y – axis is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

(ii) **Required To Find:** matrix N, which represents a clockwise rotation of 180° about the origin.
Solution:
The matrix, N, which represents a clockwise rotation of 180° about O is $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

(iii) **Required To Find:** matrix T, which represents a translation of – 3 units parallel to the x – axis and 5 units parallel to the y – axis.
Solution:
A translation of – 3 units parallel to the x – axis (3 units horizontally to the left) and 5 units parallel to the y – axis (5 units vertically upwards) may be represented by

$$T = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$$

(iv) **Data:**
$P \xrightarrow{RN} P'$, that is N first, then R second.
Required To Find: the coordinates of P' and P''.
Solution:

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 6 \\ 11 \end{pmatrix} = \begin{pmatrix} (-1\times 6)+(0\times 11) \\ (0\times 6)+(-1\times 11) \end{pmatrix}$$

$$= \begin{pmatrix} -6 \\ -11 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -6 \\ -11 \end{pmatrix} = \begin{pmatrix} (-1\times -6)+(0\times -11) \\ (0\times -6)+(1\times -11) \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -11 \end{pmatrix}$$

$$\therefore P' = (6,\ -11)$$

$$P \xrightarrow{NT} P''$$

$$\begin{pmatrix} 6 \\ 11 \end{pmatrix} \xrightarrow{T=\begin{pmatrix} -3 \\ 5 \end{pmatrix}} \begin{pmatrix} 6-3 \\ 11+5 \end{pmatrix} = \begin{pmatrix} 3 \\ 16 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 16 \end{pmatrix} \xrightarrow{N} P''$$

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ 16 \end{pmatrix} = \begin{pmatrix} (-1\times 3)+(0\times 16) \\ (0\times 3)+(-1\times 16) \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ -16 \end{pmatrix}$$

$$\therefore P'' = (-3, -16)$$

